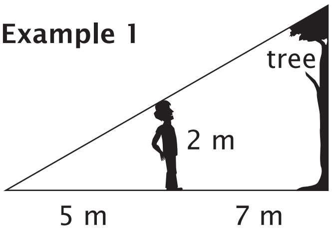
LESSON 6

Angles of Elevation and Depression

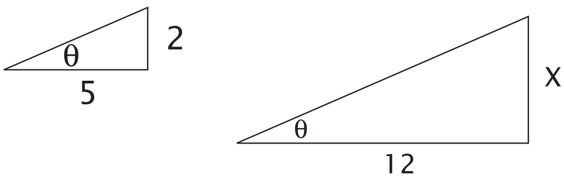
Now we get a chance to apply all of our newly acquired skills to real-life applications, otherwise known as word problems. Let's look at some elevation and depression problems. I first encountered these in a Boy Scout handbook many years ago. There was a picture of a tree, a boy, and several lines.

Example 1



How tall is the tree?

Separating the picture into two triangles helps to clarify our ratios.



We could write this as a proportion (two ratios), $\frac{2}{5} = \frac{X}{11}$, and solve for X.

We can also use our trig abilities.

From the "boy" triangle $\tan\theta = \frac{2}{5} = 0{,}4$ $\theta = 21{,}8^\circ$

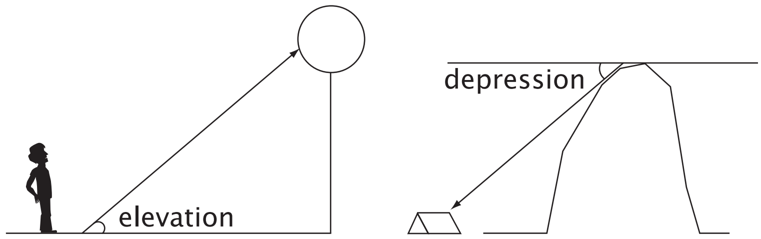
From the large triangle $\tan 21{,}8^\circ = \frac{X}{12}$

Solve for X. $(12)(0{,}4) = X$

$4{,}8 = X$ The tree is 4,8 m tall.

It is pretty obvious that an ***angle of elevation*** measures up and an ***angle of depression*** measures down. One of the keys to being a good problem solver is to draw a picture using all the data given. It turns a one-dimensional group of words into a two-dimensional picture.

Figure 1

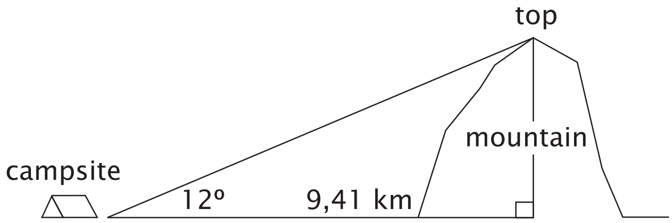


We assume that the line where the angle begins is perfectly flat or horizontal.

When working these problems, the value of the trig ratio may be rounded and recorded, and further calculations made on the rounded value. You may also keep the value of the ratio in your calculator and continue without rounding the intermediate step. This may yield slightly different final answers. These differences are not significant for the purposes of this course.

Example 2

A campsite is 9,41 km from a point directly below the mountain top. If the angle of elevation is 12º from the camp to the top of the mountain, how high is the mountain?



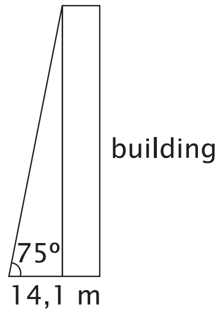
You can see a right triangle with the side adjacent to the 12º angle measuring 9,41 km. To find the height of the mountain, or the side opposite the 12º angle, the tangent is the best choice.

$$\tan 12^\circ = \frac{\text{height}}{9{,}41 \text{ km}}$$
$$(9{,}41)(\tan 12^\circ) = \text{height}$$
$$(9{,}41)(0{,}2126) = \text{height}$$
$$2 \text{ km} = \text{height}$$

Example 3

At a point 14,1 metres from the base of a building, the angle of elevation of the top is 75º. How tall is the building?

$$\tan 75^\circ = \frac{\text{height}}{14{,}1 \text{ m}}$$
$$(14{,}1)(\tan 75^\circ) = \text{height}$$
$$(14{,}1)(3{,}7321) = \text{height}$$
$$52{,}62 \text{ m} = \text{height of the building}$$



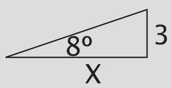
Practice Problems 1

1. How far from the door must a ramp begin in order to rise three metres with an 8º angle of elevation?

2. An A-frame cabin is 8,74 metres high at the centre, and the angle the roof makes with the base is 53º15'. How wide is the base?

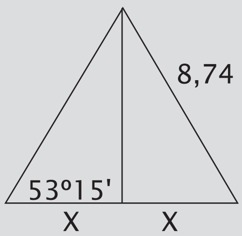
Solutions 1

1.



$$\tan 8^\circ = \frac{3}{X}$$
$$X \tan 8^\circ = 3$$
$$X = \frac{3}{\tan 8^\circ}$$
$$X = \frac{3}{0{,}1405}$$
$$X = 21{,}35 \text{ m}$$

2.



$$53^\circ 15' = 53{,}25^\circ$$
$$\tan 53{,}25^\circ = \frac{8{,}74}{X}$$
$$X = \frac{8{,}74}{\tan 53{,}25^\circ}$$
$$X = \frac{8{,}74}{1{,}3392}$$
$$X = \frac{8{,}74}{1{,}3392}$$
$$X = 6{,}53$$
$$2X = 13{,}06 \text{ m}$$