LESSON 3

Negative Numbers, Multiplication

Multiplication is fast adding of the same number. In this case, it is fast adding of a negative number. The problem (3) x (–2) is a way of writing (–2) counted three times, or (–2) + (–2) + (–2), or (–6). Think of it as borrowing $2 from someone for three days in a row. After three days you will owe $6.

Example 1 (–6)(+3) = (–18)

Example 2 (+7)(–6) = (–42)

Once multiplying a negative number by a positive number clicks, consider what you would have if you were multiplying a negative number by a negative number. It will be the opposite of what we just learned, so we are back to being positive. There are only two options for a number: either it is negative or it is positive.

Since we first learned about multiplication, we always multiplied positive numbers by positive numbers. To understand a negative number times a negative number, let's review what we know so far with several more examples.

Example 3 (+3)(+7) = (+21)

Example 4 (–3)(+7) = (–21)

Example 5 (+3)(–7) = (–21)

The only option remaining is example 6.

Example 6 $(-3)(-7) = (+21)$

Think of negative anything as the opposite of what it was. We know that two wrongs don't make a right, but when multiplying two negative numbers, the product is a positive number. Here are a several more ways of thinking of this to help us understand a difficult concept.

In language, we know that a double negative is a positive. I used to ask students if they were going to the local town fair. They would reply that they weren't not going. I would respond by saying that I would see them there. In response to their puzzled expressions I would explain that if they were "Not, not going," then they were going.

Another way to think of it is using the idea of opposites as discussed in the previous lesson. Recall that $-(-21)$ is the same as $+21$. Using brackets for clarification, I can write $(-3)(-7)$ as $-[(3)(-7)]$ by moving the negative sign in front of the 3 outside of the brackets. After multiplying $(3)(-7)$, we have (-21) inside the brackets. Then putting it all together, we have $-[-21]$, which is $(+21)$.

Example 7 $(-12)(-5) = (+60)$

Have you observed the pattern that if you have two negative signs, you are positive? The same holds for four negative signs. Whenever you have an even number of negative signs the answer is positive, and an odd number of negative signs produces a negative answer. See figure 1.

Figure 1

$(-12) = (-12)$

$-(-12) = (+12)$

$-[-(-12)] = (-12)$

$-\{-[-(-12)]\} = (+12)$