

# Honors Solutions

## Honors Lesson 1

- Begin by putting x's to show that Tyler and Madison do not like tacos. That leaves Jeff as the one who has tacos as his favorite. Since you know Jeff's favorite, you can also put x's in Jeff's row, under ice cream and steak. We are told that Madison is allergic to anything made with milk, so we can put an x across from her name, under ice cream. Now we can see that Tyler is the only one who can have ice cream as his favorite, leaving Madison with steak.

	ice cream	tacos	steak
Jeff	X	yes	X
Tyler	yes	X	X
Madison	X	X	yes

- We use similar reasoning for the rest of the problems. Remember that once you have a "yes" in any row or column, the rest of the possibilities in that row and in that column can be eliminated.

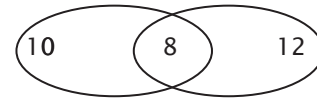
	black	brown	blonde
Mike	yes	X	X
Caitlyn	X	X	yes
Lisa	X	yes	X

- |        | reading | tennis | cooking | eating |
|--------|---------|--------|---------|--------|
| George | X       | X      | yes     | X      |
| Celia  | X       | yes    | X       | X      |
| Donna  | yes     | X      | X       | X      |
| Adam   | X       | X      | X       | yes    |

- |        | spring | summer | autumn | winter |
|--------|--------|--------|--------|--------|
| David  | X      | X      | yes    | X      |
| Linda  | X      | X      | X      | yes    |
| Shauna | yes    | X      | X      | X      |
| April  | X      | yes    | X      | X      |

## Honors Lesson 2

- $18 + 20 = 38$   
 $38 - 30 = 8$  days had both  
 Sun Rain



$$S \cap R = 8$$

$$S \cup R = 30$$

- 1 (The twisted ring you started with is called a Mobius strip.)
- 1st time : one long loop is created  
2nd time : two interlocked loops are created
- $$\left[ \frac{(5^2 + 5)}{6} \right] + 10 =$$

$$\left[ \frac{(25 + 5)}{6} \right] + 10 =$$

$$(30 \div 6) + 10 =$$

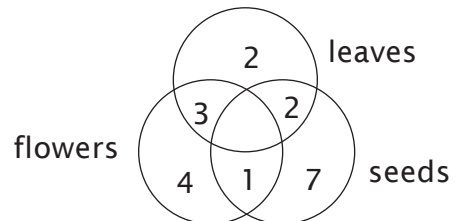
$$5 + 10 = 15$$
- $$(42 \div 7) + 6 - 1 =$$

$$(6) + 6 - 1 =$$

$$12 - 1 = 11$$

## Honors Lesson 3

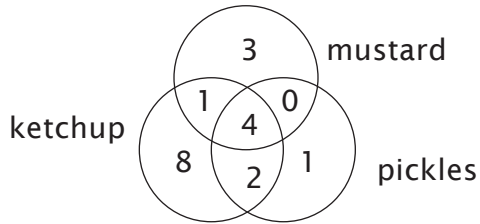
- 2
- 4
- $L \cap F = 3$
- $L \cup F = 12$



- 8
- 3

7.  $K \cup M - P = 12$

8.  $K \cap M \cap P = 4$



9.  $3 \times 4 = 4 \times 3$   
commutative property  
is true for multiplication

10.  $9 - 6 \neq 6 - 9$   
commutative property  
is false for subtraction

11.  $(2+1)+5 = 2+(1+5)$   
associative property  
is true for addition

12.  $2 \div 8 \neq 8 \div 2$   
commutative property  
is false for division

#### Honors Lesson 4

1.  $45^\circ$
2. NNW
3. NNE
4. no, he should have corrected  $67.5^\circ$
5.  $5X - 6 = 2X + 18$   
 $5X - 2X = 18 + 6$   
 $3X = 24$   
 $X = 8$
6.  $2C + 10 = 43 - C$   
 $3C = 33$   
 $C = 11$
7.  $(\$1.75 + D) + D = \$3.25$   
 $2D + \$1.75 = \$3.25$   
 $2D = \$1.50$   
 $D = \$0.75$   
 $\$.75 + \$1.75 = \$2.50$   
Drink is \$.75  
Sandwich is \$2.50

8. let  $X$  = number of Isaac's customers

$2X$  = number of Aaron's customers

$X + 2X = 105$

$3X = 105$

$X = 35$

$2X = 70$

Isaac has 35 customers

Aaron has 70 customers

9.  $X + 2X = 18$

$3X = 18$

$X = 6$  feet;  $2X = 12$  feet

10.  $A + (A + 20) = 144$

$2A + 20 = 144$

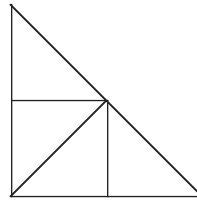
$2A = 124$

$A = 62$  apples in one box

$62 + 20 = 82$  apples in the other box

#### Honors Lesson 5

1.



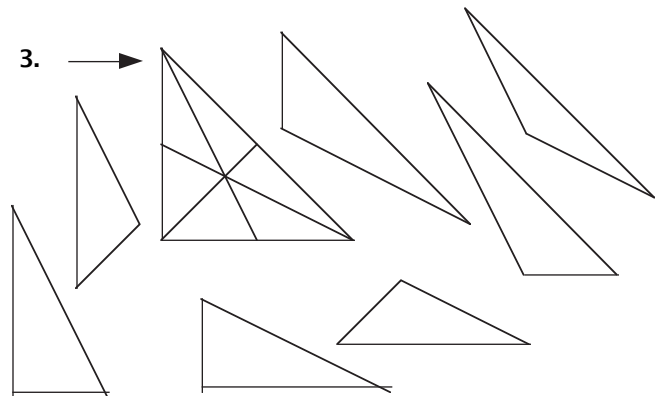
2.

4 small  
2 medium  
1 large  

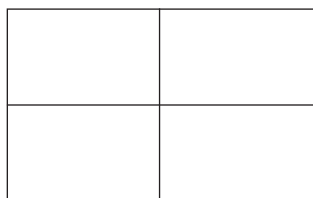
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7 total

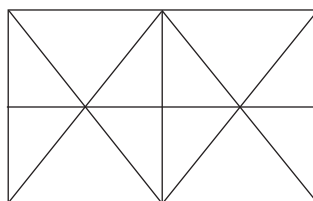
3.



4. 1 started with  
2 that are half of first triangle  
6 small  
7 overlapping (you may need to  
16 total draw these  
separately to be  
able to count each  
one. See Above.)



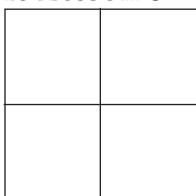
5.



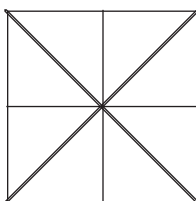
6.

### Honors Lesson 6

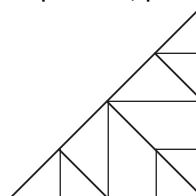
1.



2.



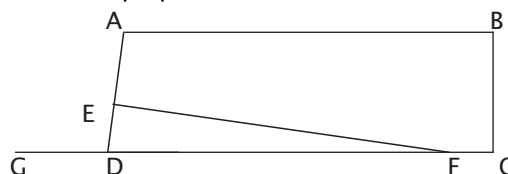
3. triangles, squares,  
trapezoids, pentagons



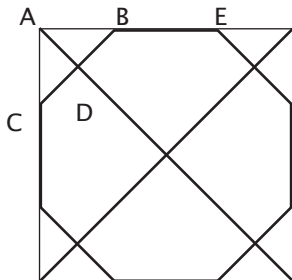
4. answers will vary  
5.  $P = 6X + .5(6)X$   
 $P = 6X + 3X$   
 $P = 9X$   
6.  $P = 9X$   
 $P = 9(8)$   
 $P = \$72$

### Honors Lesson 7

1. Extend all segments  
 $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$   
 $\overline{AB} \parallel \overline{RS} \parallel \overline{DC}$   
corresponding angles  
are congruent  
2. Yes; extend  $\overline{DF}$  and  $\overline{BC}$   
these 2 line segments are  
cut by transversal  $\overline{AB}$   
corresponding  $\angle$ 's  $\angle ADF$  and  
 $\angle ABE$  are both  $90^\circ$   
3. extend  $\overline{DC}$  to include point G  
 $m\angle A = 100^\circ$   
since  $\overline{AB}$  and  $\overline{DC}$  are parallel,  
 $m\angle GDA$  is  $100^\circ$ .  
 $m\angle EDF$  is  $80^\circ$ , since it is  
supplementary to  $\angle GDA$ .  
 $m\angle DEF = 90^\circ$  - definition  
of perpendicular



4.  $CAB = 90^\circ$  (given)  
 $BAD = 45^\circ$  - definition of bisector  
 $ADB = 90^\circ$  - definition of perpendicular  
 $ABD = 45^\circ$  - from information given  
 $DBE = 135^\circ$  - supplementary angles  
 all other corners work out the same way.



Using similar reasoning, and looking at triangles AEC, BFC, ABF, DBC and ADC, we can find the following:

$$m = 115^\circ$$

$$r = 95^\circ$$

$$f = 85^\circ$$

$$b = 110^\circ$$

$$g = 70^\circ$$

Now we know two angles from each of the smaller triangles. Armed with this knowledge, and the fact that there are  $180^\circ$  in a triangle, we can find the remaining angles:

$$c = 45^\circ$$

$$e = 70^\circ$$

$$q = 65^\circ$$

$$n = 45^\circ$$

$$k = 70^\circ$$

$$h = 65^\circ$$

You can also use what you know about vertical angles and complementary angles to find some of the angles.

## Honors Lesson 8

1. Look at the drawing below to see how the angles are labeled for easy reference.  
 $a$  and  $d$  are  $25^\circ$   
 definition of bisector

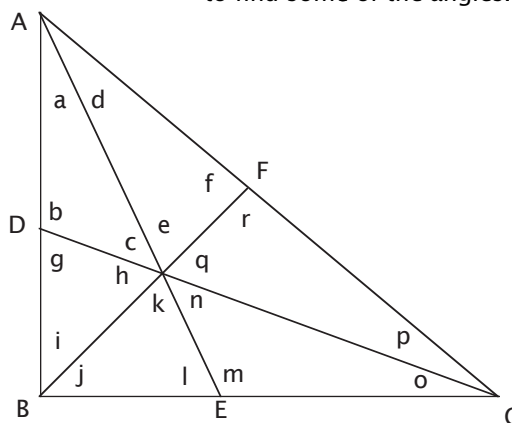
$p$  and  $o$  are  $20^\circ$   
 definition of bisector

$i$  and  $j$  are  $45^\circ$   
 definition of bisector

Now look at triangle AEB. Its angles must add up to  $180^\circ$ . We know the measure of  $a$  and that of  $ABC$ . Add these together, and subtract the result from the total  $180^\circ$  that are in a triangle:

$$180 - (25 + 90) = 180 - 115 = 65^\circ$$

$$l = 65^\circ$$



2.  $b, d, j$  and  $k$  are all  $90^\circ$  definition of perpendicular  
 $c = 180^\circ - (a + b)$   $180^\circ$  in a triangle  
 $c = 180^\circ - (60 + 90) = 30^\circ$   
 $l = 180^\circ - (k + m)$   $180^\circ$  in a triangle

$$l = 180^\circ - (90 + 30) = 60^\circ$$

$$l + i = 90^\circ$$

Angle EGC is  $90^\circ$  because of the definition of perpendicular.

$$60 + i = 90^\circ$$

$$i = 30^\circ$$

$$h = 180^\circ - (i + j)$$

$180^\circ$  in a triangle

$$h = 180^\circ - (30 + 90)$$

$$h = 60^\circ$$

$$f + h = 180^\circ$$

Angle BEC is  $180^\circ$

$$f + 60 = 180^\circ$$

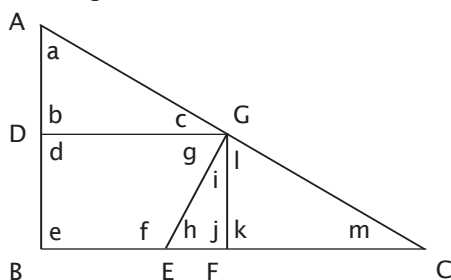
$$f = 120^\circ$$

$$c + g = 90^\circ$$

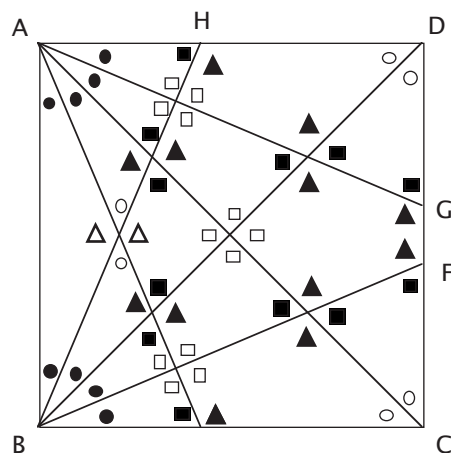
Angle AGE is  $90^\circ$  because of the definition of perpendicular.

$$30 + g = 90^\circ$$

$$g = 60^\circ$$



3. Use the same process for this one. Remember that you can also use what you know about vertical angles or complementary and supplementary angles as a shortcut.



- =  $22.5^\circ$
- =  $45^\circ$
- =  $67.5^\circ$
- =  $90^\circ$
- ▲ =  $112.5^\circ$
- △ =  $135^\circ$

## Honors Lesson 9

1. large rectangle:

$$15' 6'' = 15.5 \text{ ft}$$

$$15.5 \times 13 = 201.5 \text{ ft}^2$$

small rectangle:

$$3 \times 5 = 15 \text{ ft}^2$$

large trapezoid:

$$(9) \left( \frac{10+4}{2} \right) = (9) \left( \frac{14}{2} \right) = (9)(7) = 63 \text{ ft}^2$$

small trapezoid:

$$(2) \left( \frac{4+8}{2} \right) = (2) \left( \frac{12}{2} \right) = (2)(6) = 12 \text{ ft}^2$$

total:

$$201.5 + 15 + 63 + 12 = 291.5 \text{ ft}^2$$

2. It is necessary sometimes to add lines to the drawing to make it clearer. In figure 1a, dotted lines have been added to show how one end of the figure has been broken up. Since we know that the long measurement is 6.40 in and the space between the dotted lined is .80 in, we can see that the heights of the trapezoids add up to 5.60 in. Since we have been told that the top and bottom are the same, each trapezoid must have a height of 2.80 in.

Area of each trapezoid:

$$(2.8) \left( \frac{1.27 + .80}{2} \right) = (2.8) \left( \frac{2.07}{2} \right) =$$

$$2.898 \text{ in}^2$$

Since there are four trapezoids in all, we multiply by 4:

$$2.898 \times 4 = 11.592 \text{ in}^2$$

Rectangular center portion:

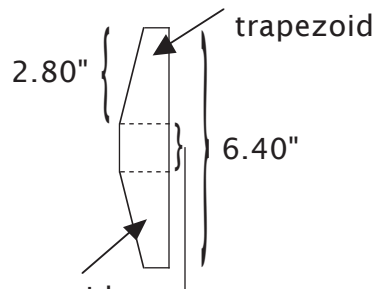
$$.80 \text{ in} \times 15 \text{ in} = 12 \text{ in}^2$$

Total:

$$12 + 11.592 = 23.592 \text{ in}^2$$

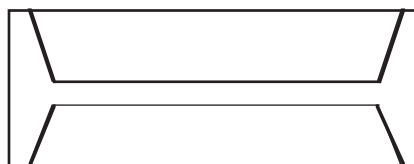
3. area = (a)(b) or ab (see figure 2)
4. area = (2a)(2b) or 4ab (see figure 3)
5. area = (na)(nb) or  $n^2 ab$  (see figure 4)
6. area =  $n^2 ab = (5^2)(4)(5) = (25)(20) = 500 \text{ ft}^2$
7. first triangle:  $a = \frac{1}{2} xy$   
 second triangle:  $a = \frac{1}{2} (2x)(2y) = 2xy$   
 4 times  $\frac{1}{2} = 2$ , so new area is four times as great.
8. first square:  $(x)(x) = x^2$   
 second square:  $(x^2)(x^2) = x^4$

figure 1a



trapezoid

figure 1b (shows a different way of finding the area)



$$\text{Area of large rectangle } 15 \times 6.4 = 96 \text{ in}^2$$

One trapezoid

long base

$$15 - (2 \times .8) = 13.4 \text{ in}^2$$

short base

$$15 - (2 \times 1.27) =$$

$$12.46 \text{ in}^2$$

$$\text{height } (6.4 - .8) \div$$

$$2 = 2.8 \text{ in}^2$$

Area of one trapezoid

$$= 36.204 \text{ in}^2$$

Both trapezoids

$$2 \times 36.204 =$$

$$72.408 \text{ in}^2$$

Area of figure

$$96 - 72.408 =$$

$$23.592 \text{ in}^2$$

figure 2

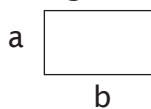
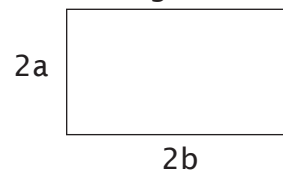
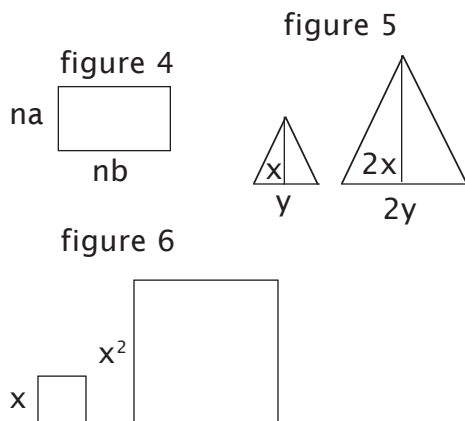


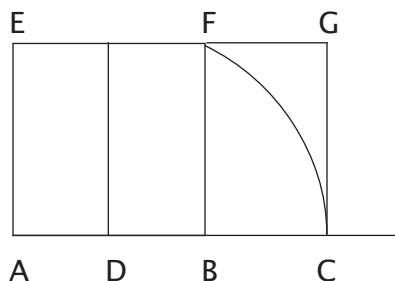
figure 3





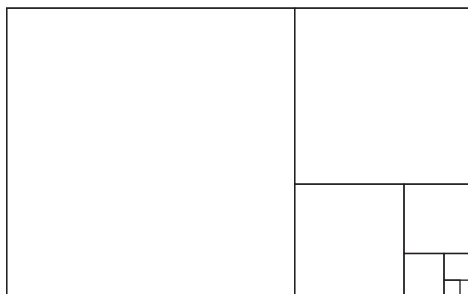
### Honors Lesson 10

1-4.



5. your answer should be close to 0.61803.
6. See illustration above.  
The ratio should be close to what you got in #5.

7-8.



### Honors Lesson 11

1.

	green, buttons	green, zipper	red, zipper	blue, buttons
Chris	yes	x	x	x
Douglas	x	yes	x	x
Ashley	x	x	x	yes
Naomi	x	x	yes	x

2.

	planning games	refreshments	place for party	birthday guest
Sam	x	x	yes	x
Jason	x	x	x	yes
Shane	yes	x	x	x
Troy	x	yes	x	x

3.

	train	boat	airplane	car
Janelle	yes	x	x	x
Walter	x	x	x	yes
Julie	x	yes	x	x
Jared	x	x	yes	x

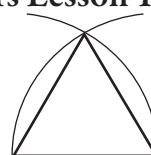
4.

	hot dog	pizza	chicken soup	tossed salad
Molly	yes	x	x	x
Tina	x	x	x	yes
Logan	x	x	yes	x
Sam	x	yes	x	x

5. Answers will vary.

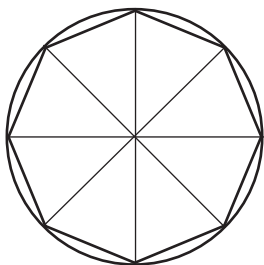
### Honors Lesson 12

1.



2.  $60^\circ$
3. Since the sections are all equal, the center angles are all the same.  $360^\circ \div 8 = 45^\circ$

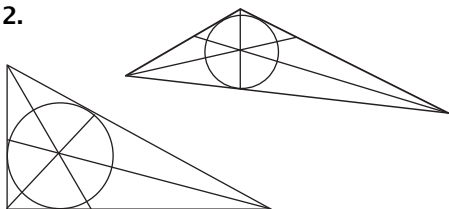
4-8.



9. In #3, you divided  $360^\circ$  by 8 to find that each small triangle has a central angle of  $45^\circ$ . Since a hexagon has six sides, you want to construct six triangles inside the circle.  $360^\circ \div 6 = 60^\circ$

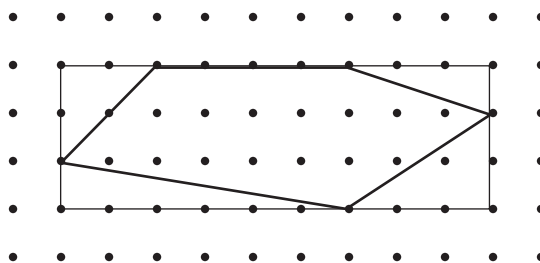
In #1, you learned how to construct an equilateral triangle with each angle equal to  $60^\circ$ . After drawing a circle and one diameter, use the same procedure to construct equilateral triangles inside your circle, using a radius of the circle as your starting point each time. After you have constructed four triangles, connect their points, and you will have an inscribed regular hexagon.

10-12.

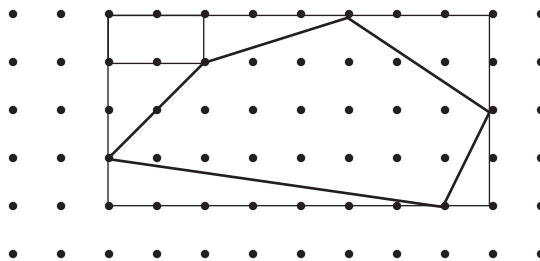


## Honors Lesson 13

1. See illustration.



2.  $3 \times 9 = 27 \text{ units}^2$
3.  $\frac{1}{2}(2 \times 2) = 2 \text{ units}^2$   
 $\frac{1}{2}(3 \times 1) = 1.5 \text{ units}^2$   
 $\frac{1}{2}(1 \times 6) = 3 \text{ units}^2$   
 $\frac{1}{2}(3 \times 2) = 3 \text{ units}^2$
4.  $2 + 1.5 + 3 + 3 = 9.5 \text{ units}^2$   
 $27 - 9.5 = 17.5 \text{ units}^2$
5. See illustration for 5 & 6.
6. See illustration for 5 & 6.



7.  $4 \times 8 = 32 \text{ units}^2$
8.  $1 \times 2 = 2 \text{ units}^2$   
 $\frac{1}{2}(1 \times 3) = 1.5 \text{ units}^2$   
 $\frac{1}{2}(2 \times 3) = 3 \text{ units}^2$   
 $\frac{1}{2}(2 \times 2) = 2 \text{ units}^2$   
 $\frac{1}{2}(1 \times 7) = 3.5 \text{ units}^2$   
 $\frac{1}{2}(1 \times 2) = 1 \text{ unit}^2$



9.  $2 + 1.5 + 3 + 2 + 3.5 + 1 = 13 \text{ units}^2$

$$32 - 13 = 19 \text{ units}^2$$

10. See illustration.

$$10 \times 5 = 50 \text{ units}^2$$

$$\frac{1}{2}(5 \times 2) = 5 \text{ units}^2$$

$$1 \times 5 = 5 \text{ units}^2$$

$$\frac{1}{2}(1 \times 5) = 2.5 \text{ units}^2$$

$$\frac{1}{2}(4 \times 1) = 2 \text{ units}^2$$

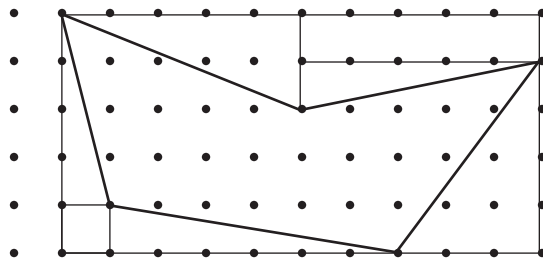
$$1 \times 1 = 1 \text{ unit}^2$$

$$\frac{1}{2}(6 \times 1) = 3 \text{ units}^2$$

$$\frac{1}{2}(3 \times 4) = 6 \text{ units}^2$$

$$5 + 5 + 2.5 + 2 + 1 + 3 + 6 = 24.5 \text{ units}^2$$

$$50 - 24.5 = 25.5 \text{ units}^2$$



11. See illustration.

$$4 \times 8 = 32 \text{ units}^2$$

$$\frac{1}{2}(1 \times 1) = .5 \text{ units}^2$$

$$\frac{1}{2}(3 \times 1) = 1.5 \text{ units}^2$$

$$1 \times 1 = 1 \text{ unit}^2$$

$$\frac{1}{2}(1 \times 2) = 1 \text{ unit}^2$$

$$\frac{1}{2}(1 \times 2) = 1 \text{ unit}^2$$

$$1 \times 1 = 1 \text{ unit}^2$$

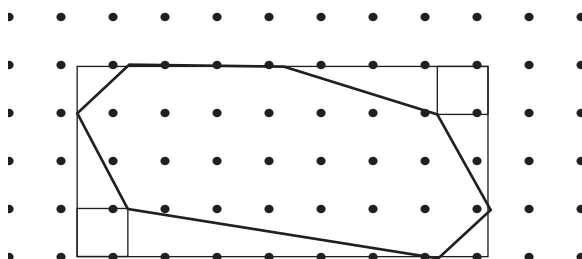
$$\frac{1}{2}(5 \times 1) = 2.5 \text{ units}^2$$

$$\frac{1}{2}(2 \times 1) = 1 \text{ unit}^2$$

$$.5 + 1.5 + 1 + 1 + 1 + 1 + 1 + 2.5 + 1$$

$$= 9.5 \text{ units}^2$$

$$32 - 9.5 = 22.5 \text{ units}^2$$



### Honors Lesson 14

1.  $\frac{1}{2}(3 \times 4) = \frac{1}{2}(12) = 6 \text{ units}^2$

2.  $A = \sqrt{s(s-a)(s-b)(s-c)}$   
 $A = \sqrt{6(6-3)(6-4)(6-5)}$   
 $A = \sqrt{6(3)(2)(1)}$   
 $A = \sqrt{36}$   
 $A = 6 \text{ units}^2$   
 yes

3.  $A = \sqrt{16(16-7)(16-10)(16-15)}$   
 $A = \sqrt{16(9)(6)(1)}$   
 $A = \sqrt{864}$   
 $A = 29.39 \text{ units}^2$
4.  $A = \sqrt{52(52-36)(52-28)(52-40)}$   
 $A = \sqrt{52(16)(24)(12)}$   
 $A = \sqrt{239,616}$   
 $A = 489.51 \text{ units}^2$
5.  $V = \pi r^2 h$   
 $V = 3.14(2)^2(10)$   
 $V = 3.14(4)(10)$   
 $V = 125.6 \text{ in}^3$
6.  $V = \pi r^2 h$   
 $V = 3.14(1)^2(10)$   
 $V = 3.14(1)(10)$   
 $V = 31.4 \text{ in}^3$   
It is  $\frac{1}{4}$  the first one
7.  $V = \pi r^2 h$   
 $V = 3.14(2)^2(5)$   
 $V = 3.14(4)(5)$   
 $V = 62.8 \text{ in}^3$   
It is half the first one.
8.  $V = \pi r^2 h$   
 $V = 3.14(4)^2(10)$   
 $V = 3.14(16)(10)$   
 $V = 502.4 \text{ in}^3$   
It is four times the first one.
9.  $V = \pi r^2 h$   
 $V = 3.14(2)^2(20)$   
 $V = 3.14(4)(20)$   
 $V = 251.2 \text{ cu in}^3$   
It is two times the first one

10. When the height is doubled, the volume is doubled. When the height is halved, the volume is halved. When the radius is doubled, the volume increases by a factor of 4. When the radius is halved, the volume decreases by a factor of 4. The student may use his own words to express this.
11. Answers will vary.
12. Take the formula, and multiply both sides by 2:  
 $V = \pi r^2 h$   
 $2V = 2\pi r^2 h$   
Now rearrange the factors:  
 $V = \pi r^2 h$   
 $2V = \pi r^2 2h$   
Take the formula, and multiply both sides by 4:  
 $V = \pi r^2 h$   
 $4V = 4\pi r^2 h$   
Rewrite the 4 on the right side as  $2^2$ :  
 $4V = 2^2 \pi r^2 h$   
Rearrange the factors:  
 $4V = \pi 2^2 r^2 h$   
 $4V = \pi (2r)^2 h$   
There is more than one way to set this up. As long as you show the same results as by experimentation, the answer is correct.

**Honors Lesson 15**

1.  $3 \times 3 \times 3 = 27 \text{ ft}$
2.  $12 \times 12 \times 12 = 1,728 \text{ in}^3$
3.  $8 \times 4 \times 2 = 64 \text{ in}^3$   
 $64 \times .3 = 19.2 \text{ lb}$
4.  $64 \text{ in}^3 \div 1,728 = .037 \text{ ft}^3$   
 $.037 \times 1200 = 44.4 \text{ lbs}$   
 You could probably lift it,  
 but it would be much heavier  
 than expected.
5. First find what the volume would  
 be if it were solid:  
 $V = \pi r^2 h$   
 $V = 3.14(.5)^2(12)$   
 $V = 9.42 \text{ in}^3$   
 Now find the volume inside  
 the pipe:  
 $V = \pi r^2 h$   
 $V = 3.14(.25)^2(12)$   
 $V = 2.355 \text{ in}^3$   
 Then find the difference:  
 $9.42 - 2.355 = 7.065 \text{ in}^3$
6.  $7.065 \times .26 = 1.8369 \text{ lb}$

7.  $V = \frac{4}{3} \pi r^3$   
 $V = \frac{4}{3} (3.14) (.25)^3$   
 $V = .07 \text{ in}^3 (\text{rounded})$   
 $.07 \times .3 \approx .02 \text{ pounds for}$   
 one bearing  
 $25 \div .02 = 1,250 \text{ bearings}$   
 Because we rounded some  
 numbers, the actual number  
 of bearings in the box may be  
 slightly different. Keep in mind  
 that the starting weight was  
 rounded to a whole number.  
 Our answer is close enough to  
 be helpful in a real life situation,  
 where someone wants to know  
 approximately how many bearings  
 are available without counting.
8. The side view is a trapezoid,  
 and the volume of the water  
 is the area of the trapezoid  
 times the width of the pool:  
 $A = \frac{3+10}{2} (40)$   
 $A = 6.5(40)$   
 $A = 260 \text{ ft}^2$   
 $V = 260(20)$   
 $V = 5,200 \text{ ft}^3$
9. Volume of the sphere:  
 $V = \frac{4}{3} (3.14) (1)^3 \text{ units}^3$   
 $V = 4.19$   
 Volume of the cube:  
 $V = 2 \times 2 \times 2 = 8 \text{ units}^3$   
 $8 - 4.19 = 3.81 \text{ units}^3$

10. Volume of the cylinder:

$$V = 3.14 (1)^2(2)$$

$$V = 6.28 \text{ units}^3$$

Volume of the sphere from #9:

$$4.19 \text{ units}^3$$

$$6.28 - 4.19 = 2.09 \text{ units}^3$$

Note: You may use the fractional value of  $\pi$  if it seems more convenient.

**Honors Lesson 16**

- $(r)\pi r = \pi r^2$
- $A = LW + LW + LH + LH + WH + WH$   
 $= 2LW + 2LH + 2WH$   
 $= 2(LW + LH + WH)$
- $2(s^2 + s^2 + s^2) = 2(3s^2) = 6s^2$
- $V = 3(11)(3) = 99 \text{ ft}^3$   
 $SA = 2(3 \times 11) + 2(3 \times 3) + 2(11 \times 3)$   
 $= 2(33) + 2(9) + 2(33)$   
 $= 66 + 18 + 66$   
 $= 150 \text{ ft}^2$
- $150 \text{ ft}^2 \div 6 \text{ faces} = 25 \text{ ft}^2 \text{ per face}$   
 $\sqrt{25} = 5 \text{ ft}$   
 The new bin is  $5 \times 5 \times 5$ .
- The cube-shaped one holds more.  
 $125 - 99 = 26 \text{ ft}^3 \text{ difference.}$

**Honors Lesson 17**

- $V = \pi r^2 h$   
 $V = 3.14(2)^2(4)$   
 $V = 50.24 \text{ ft}^3$

$$2. \quad V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}(3.14)(2)^3$$

$$V = 33.49 \text{ ft}^3 (\text{rounded})$$

$$3. \quad V = 3.14(3)^2(6)$$

$$V = 169.56 \text{ units}^3$$

$$4. \quad V = \frac{4}{3}(3.14)(3)^3$$

$$V = 113.04 \text{ units}^3 (\text{rounded})$$

$$5. \quad V = 3.14(1)^2(2)$$

$$V = 6.28 \text{ units}^3$$

$$6. \quad V = \frac{4}{3}(3.14)(1)^3$$

$$V = 4.19 \text{ units}^3 (\text{rounded})$$

$$7. \quad \frac{33.49}{50.24} \approx .67 \quad \frac{113.04}{169.56} \approx .67$$

$$\frac{4.19}{6.28} \approx .67$$

$$8. \quad \frac{2}{3}$$

$$9. \quad A = 2\pi r^2 + 2\pi rh$$

$$A = 2(3.14)(3)^2 + 2(3.14)(3)(6)$$

$$A = 56.52 + 113.04 = 169.56 \text{ units}^2$$

$$10. \quad A = 4(3.14)(3)^2$$

$$A = 113.04 \text{ units}^2$$

$$11. \quad \frac{113.04}{169.56} \approx \frac{2}{3}$$

- The surface area and volume of a sphere appear to be  $\frac{2}{3}$  of the surface area and volume of a cylinder with the same dimensions. (Archimedes proved that this is the case.)

**Honors Lesson 18**

- 4,003 mi
- $90^\circ$ ; a tangent to a circle is perpendicular to the diameter

3.  $L^2 + 4,000^2 = 4,003^2$   
 $L^2 = 4,003^2 - 4,000^2$   
 $L^2 = 16,024,009 - 16,000,000$   
 $L^2 = 24,009$   
 $L = \sqrt{24,009} \approx 155 \text{ mi}$
4.  $29,035 \div 5,280 \approx 5 \text{ mi}$
5.  $L^2 + 4,000^2 = 4,005^2$   
 $L^2 + 16,000,000 = 16,040,025$   
 $L^2 = 16,040,025 - 16,000,000$   
 $L^2 = 40,025$   
 $L = \sqrt{40,025} \approx 200 \text{ mi}$
6.  $555 \div 5,280 \approx .1$   
 $L^2 + 4,000^2 = 4,000.1^2$   
 $L^2 + 16,000,000 = 16,000,800.01$   
 $L^2 = 16,000,800.01 - 16,000,000$   
 $L^2 = 800.01$   
 $L = \sqrt{800.01} \approx 28.3 \text{ mi}$
7.  $150^2 + 4,000^2 = (X + 4,000)^2$
8.  $X^2 + 8,000X + 16,000,000$
9.  $22,500 + 16,000,000$   
 $= X^2 + 8,000X + 16,000,000$   
 $22,500 = X^2 + 8,000X$   
 $0 = X^2 + 8,000X - 22,500$   
or  $X^2 + 8,000X - 22,500 = 0$
10.  $8,000X = 22,500$   
 $X = 22,500 \div 8,000$   
 $X \approx 2.8 \text{ mi}$

## Honors Lesson 19

1.  $V = \text{area of base} \times \text{altitude}$   
 $V = (4 \cdot 4)(8)$   
 $V = 128 \text{ in}^3$
2.  $SA = 2(4 \times 4) + 2(4 \times 8) + 2(4 \times 8)$   
 $SA = 2(16) + 2(32) + 2(32)$   
 $SA = 32 + 64 + 64$   
 $SA = 160 \text{ in}^2$
3.  $V = \text{area of base} \times \text{altitude}$   
 $V = \frac{1}{2}(3 \times 4) \times 10$   
 $V = 60 \text{ ft}^3$
4.  $SA = (2)\left(\frac{1}{2}(3 \times 4)\right) + (3 \times 10) +$   
 $(4 \times 10) + (5 \times 10)$   
 $SA = 12 + 30 + 40 + 50$   
 $SA = 132 \text{ ft}^2$
5. Think of the wire as a long, skinny cylinder.  
 $1 \text{ ft}^3 = 12 \times 12 \times 12 = 1,728 \text{ in}^3$   
Volume of wire = area of base  $\times$  length  
 $1,728 = (3.14 \times .1^2) \times L$   
 $1,728 = .0314L$   
 $55,031.8 \text{ in} \approx L$   
 $55,031.8 \div 12 \approx 4,586 \text{ ft}$
6.  $A = LW$  Let  $L$  = the circumference and  $W$  = the height of the cylinder.  
Diameter = 9, so  $L = 3.14(9)$   
 $28.26 \text{ in} \approx L$  This is one dimension of the rectangle and the circumference of the cylinder.  
 $625 = 28.26W$   
 $22.12 \text{ in} = W$  This is the other dimension of the rectangle and the height of the cylinder.  
 $V = \text{area of base} \times \text{height}$   
 $V = 3.14(9 \div 2)^2 \times 22.12$   
 $V = 3.14(4.5)^2 \times 22.12$   
 $V \approx 1,406.5 \text{ in}^3$

7. Cylinder will be 4 inches high and 4 inches in diameter. Area of one circular end =  $3.14(2)^2 = 12.56 \text{ in}^2$   
 area of side =  $3.14(4) \times 4 = 50.24 \text{ in}^2$   
 $50.24 + 12.56 + 12.56 = 75.36 \text{ in}^2$   
 You also could have used what you learned in lesson 17 to find the surface area of the cylinder. First find the surface area of the sphere, and then multiply by  $\frac{3}{2}$ .  
 (See below for an alternative solution.)

7. alternative solution

$$\text{SA of sphere} = 4(3.14)(2)^2 = 50.24 \text{ in}^2$$

$$\frac{3}{2} \text{ or } 1.5(50.24) = 75.36 \text{ in}^2$$

8.  $A = 2(4 \times 4) + 2(4 \times 4) + 2(4 \times 4)$

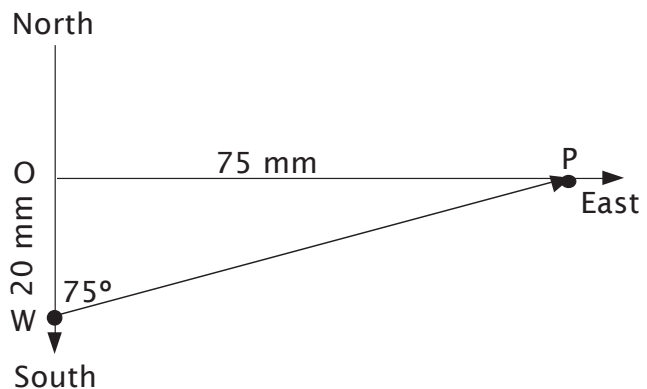
$$A = 32 + 32 + 32 = 96 \text{ in}^2$$

The cylinder uses less cardboard.

(However, there will be odd-shaped, possibly unuseable pieces left over.)

### Honors Lesson 20

- $300 \div 150 = 2$  hours
- answers may vary  
The wind blew him off course.
- $30 \div 2 = 15$  mm
- $150 \div 2 = 75$  mm
- $\angle OWP = 80^\circ$
- $\angle OWP = 75^\circ$   
 $90^\circ - 75^\circ = 15^\circ$   
 See drawing.  
 Your answers to #5 and #6 may vary slightly depending how carefully you drew and measured.



### Honors Lesson 21

- $\pi y^2$
- $A = \pi x^2 - \pi y^2$
- $y^2 + z^2 = x^2$   
 $z^2 = x^2 - y^2$
- $A = \pi(x^2 - y^2)$
- $A = \pi(z^2)$
- $A = \pi(z^2)$   
 $A = \pi\left(\frac{10}{2}\right)^2$   
 $A = \pi(5)^2$   
 $A = 3.14 \times 25$   
 $A = 78.5 \text{ in}^2$
- $A = 3.14(4)^2$   
 $A = 3.14 \times 16 = 50.24 \text{ in}^2$
- $A = L \times W$   
 $50.24 = L \times .007$   
 $50.24 \div .007 \approx 7,177 \text{ in}$
- $7,177 \div 2 \approx 3,589$  tickets  
 (rounded to the nearest whole number)

**Honors Lesson 22**

1. This bird is red.
2.  $\angle A$  is congruent to  $\angle B$ .
3. I get 100% on my math test.
4. This triangle has two congruent sides.

**Honors Lesson 23**

1. If I get burned, I touched the hot stove. Not necessarily true.
2. If two line segments are congruent, they have equal length.  
True.
3. If a bird is red, it is a cardinal.  
Not necessarily true.
4. If the leg squared plus the leg squared equals the hypotenuse squared, the triangle is a right triangle.  
True.
5. If my plants wilt, I stop watering them.  
Not true if I am sensible!

**Honors Lesson 24**

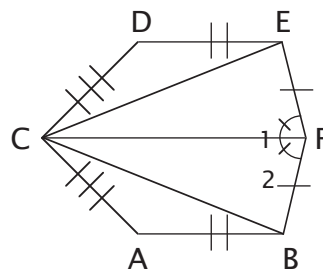
1.  $50^\circ$ ; the measure of an inscribed angle is half the measure of the intercepted arc.
2.  $130^\circ$ ;  $180^\circ - 50^\circ$
3.  $50^\circ$ ; same reason as #1
4.  $80^\circ$ ;  $180^\circ - (50^\circ + 50^\circ)$
5.  $160^\circ$ ;  $360^\circ - (100^\circ + 100^\circ)$
6.  $80^\circ$ ; vertical angles
7.  $85^\circ$ ;  $180^\circ - 95^\circ$
8.  $15^\circ$ ;  $180^\circ - (80^\circ + 85^\circ)$   
checking results with remote interior angles:  $80^\circ + 15^\circ = 95^\circ$

9.  $80^\circ$ ; angle 1 and the  $70^\circ$  angle next to it put together form an angle that is the alternate interior angle to the  $150^\circ$  angle at the top left.  
 $150^\circ - 70^\circ = 80^\circ$
10.  $70^\circ$ ; alternate interior angles
11.  $30^\circ$ ;  $180^\circ - (70^\circ + 80^\circ)$
12.  $30^\circ$ ; alternate interior angles

**Honors Lesson 25**

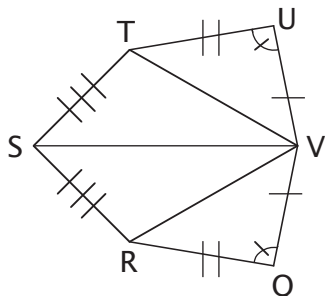
1.

Statements	Reasons
$\overline{AF} \cong \overline{EF}$	Given
$\angle 1 \cong \angle 2$	Given
$\overline{CF} \cong \overline{CF}$	Reflexive
$\triangle CEF \cong \triangle CAF$	SAS
$\overline{CE} \cong \overline{CA}$	Corresponding parts of congruent triangles
$\triangle CDE \cong \triangle CBA$	SSS
$\angle CDE \cong \angle CBA$	Corresponding parts of congruent triangles



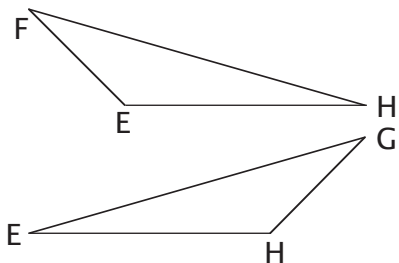
2.

Statements	Reasons
$\overline{TU} \cong \overline{RQ}$	Given
$\angle TUV \cong \angle RQV$	Given
$\overline{UV} \cong \overline{QV}$	Given
$\triangle TUV \cong \triangle RQV$	SAS
$\overline{TV} \cong \overline{RV}$	CPCTRC
$\overline{ST} \cong \overline{SR}$	Given
$\overline{SV} \cong \overline{SV}$	Reflexive
$\triangle TSV \cong \triangle RSV$	SSS
$\angle TSV \cong \angle RSV$	CPCTRC



3.

Statements	Reasons
$\overline{FE} \cong \overline{GH}$	Given
$\overline{FH} \cong \overline{GE}$	Given
$\overline{EH} \cong \overline{EH}$	Reflexive
$\triangle FEH \cong \triangle GHE$	SSS



## Honors Lesson 26

1.

Statements	Reasons
$\overline{AB} \cong \overline{AC}$	Given
$\angle ARB \cong \angle AQC$	Perpendicular
$\angle BAR \cong \angle CAQ$	Reflexive
$\triangle BAR \cong \triangle CAQ$	AAS or HA
$\overline{CQ} \cong \overline{BR}$	CPCTRC

2.

Statements	Reasons
$\overline{XB} \cong \overline{YB}$	Definition of bisector
$\angle XBA \cong \angle YBA$	Definition of Perpendicular
$\overline{BA} \cong \overline{BA}$	Reflexive
$\triangle XBA \cong \triangle YBA$	SAS or LL
$\overline{XA} \cong \overline{YA}$	CPCTRC

3.

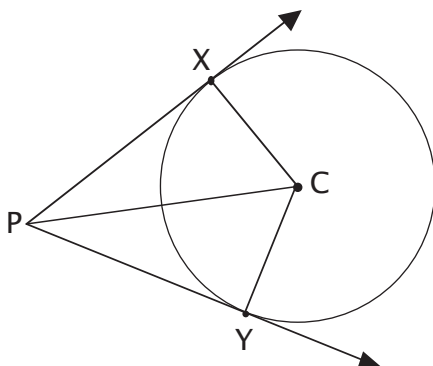
Statements	Reasons
$\overline{EF} \cong \overline{GF}$	From proof above
$\overline{EX} \cong \overline{GX}$	Definition of Bisector
$\overline{FX} \cong \overline{FX}$	Reflexive
$\triangle EFX \cong \triangle GFX$	SSS or HL
$\angle EXH \cong \angle GXH$	Definition of Perpendicular
$\overline{HX} \cong \overline{HX}$	Reflexive
$\triangle EHX \cong \triangle GHX$	SAS or LL
$\overline{EH} \cong \overline{GH}$	CPCTRC

## Honors Lesson 27

1.

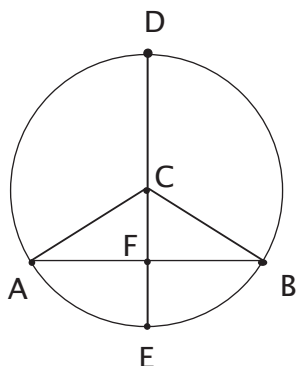
Statements	Reasons
$\overline{XC} \cong \overline{YC}$	Radius of a circle
$\angle PYC \cong \angle PXC$	A tangent of a circle is perpendicular to the radius at that point.
$\overline{PC} \cong \overline{PC}$	Reflexive
$\triangle PYC \cong \triangle PXC$	HL
$\overline{PX} \cong \overline{PY}$	CPCTRC





2.

Statements	Reasons
$\overline{DE} \perp \overline{AB}$	Given
$\overline{AC} \cong \overline{BC}$	Radius of a circle
$\overline{FC} \cong \overline{FC}$	Reflexive
$\triangle FCA \cong \triangle FCB$	HL
$\angle ACE \cong \angle BCE$	CPCTRC
$\widehat{AE} \cong \widehat{BE}$	Property of central angle



3.

Statements	Reasons
$\overline{OP} \cong \overline{LM}$	Given
$\overline{OC} \cong \overline{LC}$	Radius of a circle
$\overline{PC} \cong \overline{MC}$	Radius of a circle
$\triangle CPO \cong \triangle CML$	SSS
$\overline{OX} \cong \overline{LY}$	Definition of Bisector
$\triangle OCX \cong \triangle LCY$	HL
$\overline{XC} \cong \overline{YC}$	CPCTRC

## Honors Lesson 28

- $67 = \left(\frac{1}{2}\right)X$   
 $134^\circ = X$   
 $50\left(\frac{1}{2}\right) = Y$   
 $100^\circ = Y$   
 $180 - (50 + 67) = \left(\frac{1}{2}\right)Z$   
 $63 = \left(\frac{1}{2}\right)Z$   
 $126^\circ = Z$
- $B = 180 - 77 = 103^\circ$   
 $A = 180 - 84 = 96^\circ$   
 $C = 2 \times 77 = 154^\circ$
- $m\widehat{QR} = 2(63^\circ) = 126^\circ$   
 $m\angle QCR = m\widehat{QR} = 126^\circ$
- $m\angle AEC = \frac{40^\circ + 30^\circ}{2} = \frac{70^\circ}{2} = 35^\circ$   
 $m\angle BED = 35^\circ$
- $m\angle KPL = \frac{116^\circ - 36^\circ}{2} = \frac{80^\circ}{2} = 40^\circ$

## Honors Lesson 29

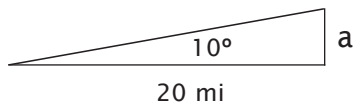
1.	angle	tan
	$10^\circ$	.18
	$15^\circ$	.268
	$30^\circ$	.58
	$45^\circ$	1
	$60^\circ$	1.73

$$2. \quad \tan 10^\circ = \frac{a}{20}$$

$$.18 = \frac{a}{20}$$

$$a = .18(20)$$

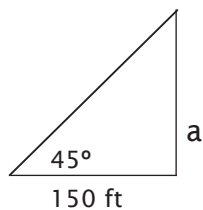
$$a = 3.6 \text{ mi or } 19,008 \text{ ft}$$



$$3. \quad \tan 45^\circ = \frac{a}{150}$$

$$1 = \frac{a}{150}$$

$$150 \text{ ft} = a$$

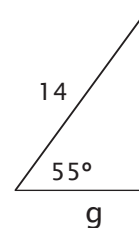


$$4. \quad \cos 55^\circ = \frac{g}{14}$$

$$.5736 = \frac{g}{14}$$

$$(14).5736 = g$$

$$g = 8.0304 \text{ ft}$$

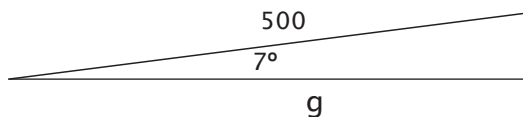


$$5. \quad \cos 7^\circ = \frac{g}{500}$$

$$.9925 = \frac{g}{500}$$

$$(500).9925 = g$$

$$g = 496.25 \text{ ft}$$



$$6. \quad \cos 60^\circ = \frac{30}{L}$$

$$.5 = \frac{30}{L}$$

$$.5L = 30$$

$$L = 60 \text{ ft}$$



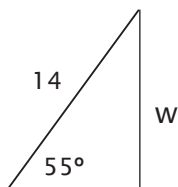
### Honors Lesson 30

$$1. \quad \sin 55^\circ = \frac{w}{14}$$

$$.8192 = \frac{w}{14}$$

$$(14).8192 = w$$

$$w = 11.4688 \text{ ft}$$



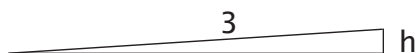
$$2. \quad \sin 4^\circ = \frac{h}{3}$$

$$.0698 = \frac{h}{3}$$

$$(3).0698 = h$$

$$h = .2094 \text{ mi}$$

$$.2094(5,280) = 1,105.632 \text{ ft}$$



$$3. \quad \sin 34^\circ = \frac{p}{4.5}$$

$$.5592 = \frac{p}{4.5}$$

$$(4.5).5592 = p$$

$$p = 2.5164 \text{ mi or } 13,286.592 \text{ ft}$$

