

$$\begin{aligned}
 8. \quad & \left(\frac{3+3\sqrt{5}}{2} \right)^2 - 3 \left(\frac{3+3\sqrt{5}}{2} \right) - 9 = 0 \\
 & \frac{9+18\sqrt{5}+9(5)}{4} - \frac{9+9\sqrt{5}}{2} - 9 = 0 \\
 & \frac{9+18\sqrt{5}+45}{4} - \frac{9+9\sqrt{5}}{2} - 9 = 0 \\
 & \frac{54+18\sqrt{5}}{4} - \frac{9+9\sqrt{5}}{2} - 9 = 0 \\
 & \frac{2(27+9\sqrt{5})}{2(2)} - \frac{9+9\sqrt{5}}{2} - 9 = 0 \\
 & \frac{27+9\sqrt{5}}{2} - \frac{9+9\sqrt{5}}{2} - \frac{18}{2} = 0 \\
 & \frac{27+9\sqrt{5}-(9+9\sqrt{5})-18}{2} = 0 \\
 & \frac{\cancel{27}+9\sqrt{5}-\cancel{9}-9\sqrt{5}-18}{2} = 0 \\
 & \frac{27-9-18}{2} = 0 \\
 & \frac{0}{2} = 0 \\
 & 0 = 0
 \end{aligned}$$

The check for the other solution is not shown here, but it works out in a similar fashion.

$$\begin{aligned}
 9. \quad & 2X^2 + 3X - 2 = 0 \\
 & (2X-1)(X+2) = 0 \\
 & 2X-1=0 \quad X+2=0 \\
 & 2X=1 \quad X=-2 \\
 & X=\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 2 = 0 \\
 & 2\left(\frac{1}{4}\right) + \frac{3}{2} - 2 = 0 \\
 & \frac{2}{4} + \frac{3}{2} - 2 = 0 \\
 & \frac{1}{2} + \frac{3}{2} - \frac{4}{2} = 0 \\
 & 0 = 0 \\
 & 2(-2)^2 + 3(-2) - 2 = 0 \\
 & 2(4) - 6 - 2 = 0 \\
 & 8 - 6 - 2 = 0 \\
 & 0 = 0
 \end{aligned}$$

$$\begin{aligned}
 11. \quad & (X+2)^5 = 1X^5 2^0 + 5X^4 2^1 + 10X^3 2^2 + \\
 & 10X^2 2^3 + 5X 2^4 + 1X^0 2^5 = \\
 & X^5 + 10X^4 + 40X^3 + 80X^2 + 80X + 32
 \end{aligned}$$

$$\begin{aligned}
 12. \quad & (2X-1)^4 = 1(2X)^4(-1)^0 + 4(2X)^3(-1)^1 + \\
 & 6(2X)^2(-1)^2 + 4(2X)^1(-1)^3 + 1(2X)^0(-1)^4 = \\
 & 16X^4 - 32X^3 + 24X^2 - 8X + 1 \\
 13. \quad & \frac{6 \cdot 5}{1 \cdot 2} (X)^4 (-1)^2 = 15X^4 \\
 14. \quad & \frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} (X)^3 (-1)^3 = -20X^3 \\
 15. \quad & (3X+1)^3 = \\
 & 1(3X)^3 1^0 + 3(3X)^2 1^1 + 3(3X)^1 1^2 + 1(3X)^0 1^3 = \\
 & 27X^3 + 27X^2 + 9X + 1 \\
 16. \quad & (X+5) \\
 17. \quad & \frac{3-2\sqrt{-5}}{7i+2} = \frac{(3-2\sqrt{-5})(7i-2)}{(7i+2)(7i-2)} = \\
 & \frac{21i-6-14i\sqrt{-5}+4\sqrt{-5}}{49i^2-4} = \\
 & \frac{21i-6-14i^2\sqrt{5}+4i\sqrt{5}}{-49-4} = \\
 & \frac{21i-6-14(-1)\sqrt{5}+4i\sqrt{5}}{-53} = \\
 & \frac{21i-6+14\sqrt{5}+4i\sqrt{5}}{-53} \\
 18. \quad & \frac{1+\sqrt{X}}{2-\sqrt{X}} = \frac{(1+\sqrt{X})(2+\sqrt{X})}{(2-\sqrt{X})(2+\sqrt{X})} = \\
 & \frac{2+\sqrt{X}+2\sqrt{X}+X}{4-X} = \frac{2+3\sqrt{X}+X}{4-X} \\
 19. \quad & (18i)(\sqrt{-36}+7i) = (18i)(6i+7i) = \\
 & (18i)(13i) = 234i^2 = 234(-1) = -234 \\
 20. \quad & (i^2)(i^1)(i^3) = i^{2+1+3} = i^6 = (-1)^3 = -1
 \end{aligned}$$

Lesson Practice 12A

$$\begin{aligned}
 1. \quad & X = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)} = \frac{-6 \pm \sqrt{28}}{2} = \\
 & \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7} \\
 2. \quad & X^2 - 5X + 4 = 0 \\
 & (X-4)(X-1) = 0 \\
 & X-4=0 \quad X-1=0 \\
 & X=4 \quad X=1 \\
 3. \quad & X = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(-1)}}{2(3)} = \frac{-7 \pm \sqrt{61}}{6} \\
 4. \quad & A^2 - 10A - 11 = 0 \\
 & (A-11)(A+1) = 0 \\
 & A-11=0 \quad A+1=0 \\
 & A=11 \quad A=-1
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & 2Q^2 + 2 = 17Q \\
 & 2Q^2 - 17Q + 2 = 0 \\
 & \frac{-(-17) \pm \sqrt{(-17)^2 - 4(2)(2)}}{2(2)} = \frac{17 \pm \sqrt{273}}{4}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & 5X^2 + 15X + 10 = 0 \\
 & (5)(X+1)(X+2) = 0 \\
 & X+1 = 0 \quad X+2 = 0 \\
 & X = -1 \quad X = -2
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & \frac{1}{4}R^2 - \frac{1}{2}R + \frac{3}{2} = 0 \\
 & (4)\frac{1}{4}R^2 - (4)\frac{1}{2}R + (4)\frac{3}{2} = (4)0 \\
 & R^2 - 2R + 6 = 0 \\
 & \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(6)}}{2(1)} = \\
 & \frac{2 \pm \sqrt{-20}}{2} = \frac{2 \pm 2i\sqrt{5}}{2} = 1 \pm i\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & 16X^2 = 2X + 4 \\
 & 8X^2 = X + 2 \\
 & 8X^2 - X - 2 = 0 \\
 & X = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-2)}}{2(8)} = \frac{1 \pm \sqrt{65}}{16}
 \end{aligned}$$

$$9. \quad X = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-8)}}{2(2)} = \frac{-3 \pm \sqrt{73}}{4}$$

$$\begin{aligned}
 10. \quad & Y^2 = \frac{3}{4}Y + 2 \\
 & (4)Y^2 = (4)\frac{3}{4}Y + (4)2 \\
 & 4Y^2 = 3Y + 8 \\
 & 4Y^2 - 3Y - 8 = 0 \\
 & X = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-8)}}{2(4)} = \frac{3 \pm \sqrt{137}}{8}
 \end{aligned}$$

Lesson Practice 12B

$$1. \quad X = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-3)}}{2(8)} = \frac{1 \pm \sqrt{97}}{16}$$

$$\begin{aligned}
 2. \quad & 7 = 2X^2 + X \\
 & 0 = 2X^2 + X - 7 \\
 & X = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-7)}}{2(2)} = \frac{-1 \pm \sqrt{57}}{4}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & Q = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)} = \frac{6 \pm \sqrt{24}}{2} = \\
 & \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & 2 + 3X + 4X^2 = 0 \\
 & 4X^2 + 3X + 2 = 0 \\
 & X = \frac{-(3) \pm \sqrt{(3)^2 - 4(4)(2)}}{2(4)} = \\
 & \frac{-3 \pm \sqrt{-23}}{8} = \frac{-3 \pm i\sqrt{23}}{8}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & P = P^2 - 2 \\
 & 0 = P^2 - P - 2 \\
 & 0 = (P-2)(P+1) \\
 & P-2 = 0 \quad P+1 = 0 \\
 & P = 2 \quad P = -1
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & X^2 + \frac{1}{5}X + 5 = 0 \\
 & (5)X^2 + (5)\frac{1}{5}X + (5)5 = (5)0 \\
 & 5X^2 + X + 25 = 0 \\
 & X = \frac{-(1) \pm \sqrt{(1)^2 - 4(5)(25)}}{2(5)} = \frac{-1 \pm i\sqrt{499}}{10}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & 20X^2 + 40X = 30 \\
 & 2X^2 + 4X = 3 \\
 & 2X^2 + 4X - 3 = 0 \\
 & X = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{40}}{4} = \\
 & \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & X = \frac{-(2) \pm \sqrt{(2)^2 - 4(5)(-1)}}{2(5)} = \frac{-2 \pm \sqrt{24}}{10} = \\
 & \frac{-2 \pm 2\sqrt{6}}{10} = \frac{-1 \pm \sqrt{6}}{5}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & 3X^2 = -5X \\
 & 3X^2 + 5X = 0 \\
 & (X)(3X+5) = 0
 \end{aligned}$$

$$\begin{aligned}
 & X = 0 \quad 3X+5 = 0 \\
 & \quad \quad 3X = -5 \\
 & \quad \quad X = -\frac{5}{3}
 \end{aligned}$$

$$10. \quad \frac{-(B) \pm \sqrt{(B)^2 - 4(A)(C)}}{2(A)} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Systematic Review 12C

1. $x^2 - 5x + 6 = 0$
 $(x-3)(x-2) = 0$
 $x-3 = 0 \quad x-2 = 0$
 $x = 3 \quad x = 2$
2. $x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(2)}}{2(1)} =$
 $\frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$
3. $x^2 - 3x + 1 = -6x$
 $x^2 + 3x + 1 = 0$
 $x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$
4. $x^2 + 4x - 12 = 0$
 $(x+6)(x-2) = 0$
 $x+6 = 0 \quad x-2 = 0$
 $x = -6 \quad x = 2$
5. $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(5)}}{2(2)} =$
 $\frac{-2 \pm \sqrt{-36}}{4} = \frac{-2 \pm 6i}{4} = \frac{-1 \pm 3i}{2}$
6. $x^2 + 8x = -16$
 $x^2 + 8x + 16 = 0$
 $(x+4)(x+4) = 0$
 $x+4 = 0 \quad x = -4$
7. 169
8. $2x^2 + 9x + \text{---}$
divide through by 2:
 $x^2 + \frac{9}{2}x + \text{---}$
complete the square:
 $x^2 + \frac{9}{2}x + \frac{81}{16}$
multiply through by 2:
 $2x^2 + 9x + \frac{81}{8}$
9. 40x
10. $2\sqrt{14}x$
11. $x^2 + \frac{1}{3}x - \frac{4}{3} = 0$
 $x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{4}{3} + \frac{1}{36}$
 $\left(x + \frac{1}{6}\right)^2 = \frac{4(12)}{3(12)} + \frac{1}{36}$
 $\left(x + \frac{1}{6}\right)^2 = \frac{49}{36}$
 $x + \frac{1}{6} = \sqrt{\frac{49}{36}}$
 $x = -\frac{1}{6} \pm \frac{7}{6}$
 $x = -\frac{1}{6} + \frac{7}{6} \quad x = -\frac{1}{6} - \frac{7}{6}$
 $x = \frac{6}{6} = 1 \quad x = -\frac{8}{6} = -\frac{4}{3}$
12. $(1)^2 + \frac{1}{3}(1) - \frac{4}{3} = 0$
 $1 + \frac{1}{3} - \frac{4}{3} = 0$
 $\frac{3}{3} + \frac{1}{3} - \frac{4}{3} = 0$
 $0 = 0$
 $\left(-\frac{4}{3}\right)^2 + \frac{1}{3}\left(-\frac{4}{3}\right) - \frac{4}{3} = 0$
 $\frac{16}{9} - \frac{4}{9} - \frac{12}{9} = 0$
 $0 = 0$
13. $(x-A)^6 = 1x^6(-A)^0 + 6x^5(-A)^1 +$
 $15x^4(-A)^2 + 20x^3(-A)^3 + 15x^2(-A)^4 +$
 $6x^1(-A)^5 + 1x^0(-A)^6 =$
 $x^6 - 6x^5A + 15x^4A^2 - 20x^3A^3 +$
 $15x^2A^4 - 6xA^5 + A^6$
14. $\frac{4}{1}\left(\frac{1}{2}x\right)^3(-3A)^1 = 4\left(\frac{1}{8}\right)x^3(-3A) = -\frac{3}{2}x^3A$
15. $(5-2A)^3 =$
 $1(5)^3(-2A)^0 + 3(5)^2(-2A)^1 +$
 $3(5)^1(-2A)^2 + 1(5)^0(-2A)^3 =$
 $125 - 150A + 60A^2 - 8A^3$
16. $(x-2y)$
17. $\frac{6+5i}{3i-2} = \frac{(6+5i)(3i+2)}{(3i-2)(3i+2)} =$
 $\frac{18i+12+15i^2+10i}{9i^2-4} = \frac{28i+12+15(-1)}{9(-1)-4} =$
 $\frac{28i+12-15}{-9-4} = \frac{28i-3}{-13}$

$$18. \frac{2+\sqrt{-49}}{2-\sqrt{-49}} = \frac{(2+\sqrt{-49})(2+\sqrt{-49})}{(2-\sqrt{-49})(2+\sqrt{-49})} = \frac{4+4\sqrt{-49}-49}{4-(-49)} = \frac{-45+4i(7)}{4+49} = \frac{-45+28i}{53}$$

$$19. \frac{2}{3-\sqrt{7}} = \frac{2(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} = \frac{6+2\sqrt{7}}{9-7} = \frac{6+2\sqrt{7}}{2} = \frac{3+\sqrt{7}}{1} = 3+\sqrt{7}$$

$$20. \frac{2+\sqrt{5}}{2\sqrt{5}-4} = \frac{(2+\sqrt{5})(2\sqrt{5}+4)}{(2\sqrt{5}-4)(2\sqrt{5}+4)} = \frac{4\sqrt{5}+8+4\sqrt{5}+2\sqrt{25}}{4(5)-16} = \frac{8\sqrt{5}+8+2(5)}{20-16} = \frac{8\sqrt{5}+8+10}{4} = \frac{8\sqrt{5}+18}{4} = \frac{4\sqrt{5}+9}{2}$$

Systematic Review 12D

$$1. x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-7)}}{2(2)} = \frac{9 \pm \sqrt{137}}{4}$$

$$2. x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-2)}}{2(1)} = \frac{-5 \pm \sqrt{33}}{2}$$

$$3. \begin{aligned} 3x^2 + 7x + 4 &= 0 \\ (3x+4)(x+1) &= 0 \\ 3x+4 &= 0 & x+1 &= 0 \\ 3x &= -4 & x &= -1 \\ x &= -\frac{4}{3} \end{aligned}$$

$$4. x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(12)}}{2(1)} = \frac{6 \pm \sqrt{-12}}{2} = \frac{6 \pm 2i\sqrt{3}}{2} = 3 \pm i\sqrt{3}$$

$$5. \begin{aligned} 5x^2 - 3x - 2 &= 0 \\ (5x+2)(x-1) &= 0 \\ 5x+2 &= 0 & x-1 &= 0 \\ 5x &= -2 & x &= 1 \\ x &= -\frac{2}{5} \end{aligned}$$

$$6. \begin{aligned} 4x^2 + 1 &= 4x \\ 4x^2 - 4x + 1 &= 0 \\ (2x-1)(2x-1) &= 0 \\ 2x-1 &= 0 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

$$7. \frac{25}{4}$$

$$8. \frac{1}{16}$$

$$9. 25x^2 + \underline{\quad} + 1$$

divide through by 25:

$$x^2 + \underline{\quad} + \frac{1}{25}$$

complete the square:

$$x^2 + \frac{2}{5}x + \frac{1}{25}$$

multiply through by 25:

$$25x^2 + \underline{10x} + 1$$

$$10. 49x^2 - \underline{\quad} + 4$$

divide through by 49:

$$x^2 - \underline{\quad} + \frac{4}{49}$$

complete the square:

$$x^2 - \frac{4}{7}x + \frac{4}{49}$$

multiply through by 49:

$$49x^2 - \underline{28x} + 4$$

$$11. \begin{aligned} x^2 - 12x + 20 &= 0 \\ (x-10)(x-2) &= 0 \end{aligned}$$

$$\begin{aligned} x-10 &= 0 & x-2 &= 0 \\ x &= 10 & x &= 2 \end{aligned}$$

$$12. \begin{aligned} (10)^2 - 12(10) + 20 &= 0 \\ 100 - 120 + 20 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} (2)^2 - 12(2) + 20 &= 0 \\ 4 - 24 + 20 &= 0 \\ 0 &= 0 \end{aligned}$$

$$13. \begin{aligned} (x+1)^4 &= \\ 1x^4 + 4x^3 + 6x^2 + 4x + 1 &= \\ x^4 + 4x^3 + 6x^2 + 4x + 1 &= \end{aligned}$$

$$14. \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4} \left(\frac{1}{2}x \right)^0 (-3A)^4 = 81A^4$$

$$15. \begin{aligned} \left(10 - \frac{1}{x} \right)^3 &= 1(10)^3 \left(-\frac{1}{x} \right)^0 + \\ 3(10)^2 \left(-\frac{1}{x} \right)^1 &+ 3(10)^1 \left(-\frac{1}{x} \right)^2 + 1(10)^0 \left(-\frac{1}{x} \right)^3 = \\ 1000 - \frac{300}{x} &+ \frac{30}{x^2} - \frac{1}{x^3} \end{aligned}$$

$$16. (x+2)$$

$$17. \frac{4-3i}{2i} = \frac{(4-3i)(i)}{2i(i)} = \frac{4i-3i^2}{2i^2} =$$

$$\frac{4i-3(-1)}{2(-1)} = \frac{4i+3}{-2}$$

$$\begin{aligned}
 18. \quad \frac{10 + \sqrt{-A}}{10 - \sqrt{-A}} &= \frac{(10 + \sqrt{-A})(10 + \sqrt{-A})}{(10 - \sqrt{-A})(10 + \sqrt{-A})} = \\
 &= \frac{100 + 20\sqrt{-A} - A}{100 - (-A)} = \frac{100 + 20i\sqrt{A} - A}{100 + A} \\
 19. \quad \frac{9}{7 + \sqrt{10}} &= \frac{9(7 - \sqrt{10})}{(7 + \sqrt{10})(7 - \sqrt{10})} = \\
 &= \frac{63 - 9\sqrt{10}}{49 - 10} = \frac{63 - 9\sqrt{10}}{39} = \frac{21 - 3\sqrt{10}}{13} \\
 20. \quad \frac{4 - \sqrt{6}}{3\sqrt{7} + 5} &= \frac{(4 - \sqrt{6})(3\sqrt{7} - 5)}{(3\sqrt{7} + 5)(3\sqrt{7} - 5)} = \\
 &= \frac{12\sqrt{7} - 20 - 3\sqrt{42} + 5\sqrt{6}}{9(7) - 25} = \\
 &= \frac{12\sqrt{7} - 20 - 3\sqrt{42} + 5\sqrt{6}}{63 - 25} = \\
 &= \frac{12\sqrt{7} - 20 - 3\sqrt{42} + 5\sqrt{6}}{38}
 \end{aligned}$$

Systematic Review 12E

$$\begin{aligned}
 1. \quad & x^2 + 2x - 8 = 0 \\
 & (x + 4)(x - 2) = 0 \\
 & x + 4 = 0 \quad x - 2 = 0 \\
 & x = -4 \quad x = 2 \\
 2. \quad & x^2 - 6x = -8 \\
 & x^2 - 6x + 8 = 0 \\
 & (x - 4)(x - 2) = 0 \\
 & x - 4 = 0 \quad x - 2 = 0 \\
 & x = 4 \quad x = 2 \\
 3. \quad & 2x^2 - 15x + 7 = 0 \\
 & (2x - 1)(x - 7) = 0 \\
 & 2x - 1 = 0 \quad x - 7 = 0 \\
 & 2x = 1 \quad x = 7 \\
 & x = \frac{1}{2} \\
 4. \quad & 3x^2 + 4x = 7 \\
 & 3x^2 + 4x - 7 = 0 \\
 & (3x + 7)(x - 1) = 0 \\
 & 3x + 7 = 0 \quad x - 1 = 0 \\
 & 3x = -7 \quad x = 1 \\
 & x = -\frac{7}{3} \\
 5. \quad & 2 = 5x + x^2 \\
 & 0 = x^2 + 5x - 2 \\
 & x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-2)}}{2(1)} = \frac{-5 \pm \sqrt{33}}{2}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & x^2 + 2x - 15 = 0 \\
 & (x + 5)(x - 3) = 0 \\
 & x + 5 = 0 \quad x - 3 = 0 \\
 & x = -5 \quad x = 3 \\
 7. \quad & 4x^2 + 28x + \underline{\quad} \\
 & \text{divide through by 4:} \\
 & x^2 + 7x + \underline{\quad} \\
 & \text{complete the square:} \\
 & x^2 + 7x + \frac{49}{4} \\
 & \text{multiply through by 4:} \\
 & 4x^2 + 28x + 49 \\
 8. \quad & 9x^2 - 36x + \underline{\quad} \\
 & \text{divide through by 9:} \\
 & x^2 - 4x + \underline{\quad} \\
 & \text{complete the square:} \\
 & x^2 - 4x + \underline{4} \\
 & \text{multiply through by 9:} \\
 & 9x^2 - 36x + \underline{36} \\
 9. \quad & 36x^2 + \underline{\quad} + 25 \\
 & \text{divide through by 36:} \\
 & x^2 + \underline{\quad} + \frac{25}{36} \\
 & \text{complete the square:} \\
 & x^2 + \frac{10}{6}x + \frac{25}{36} \\
 & \text{multiply through by 36:} \\
 & 36x^2 + \underline{60x} + 25 \\
 10. \quad & 81x^2 - \underline{\quad} + 121 \\
 & \text{divide through by 81:} \\
 & x^2 - \underline{\quad} + \frac{121}{81} \\
 & \text{complete the square:} \\
 & x^2 - \frac{22}{9}x + \frac{121}{81} \\
 & \text{multiply through by 81:} \\
 & 81x^2 - \underline{198x} + 121 \\
 11. \quad & x^2 + 5x - 14 = 0 \\
 & (x + 7)(x - 2) = 0 \\
 & x + 7 = 0 \quad x - 2 = 0 \\
 & x = -7 \quad x = 2
 \end{aligned}$$

$$\begin{aligned}
 12. \quad (-7)^2 + 5(-7) - 14 &= 0 \\
 49 - 35 - 14 &= 0 \\
 0 &= 0
 \end{aligned}$$

$$\begin{aligned}
 (2)^2 + 5(2) - 14 &= 0 \\
 4 + 10 - 14 &= 0 \\
 0 &= 0
 \end{aligned}$$

$$\begin{aligned}
 13. \quad (2X+1)^5 &= \\
 1(2X)^5 1^0 + 5(2X)^4 1^1 + 10(2X)^3 1^2 + \\
 10(2X)^2 1^3 + 5(2X)^1 1^4 + 1(2X)^0 1^5 &= \\
 32X^5 + 80X^4 + 80X^3 + 40X^2 + 10X + 1
 \end{aligned}$$

$$14. \quad \frac{5 \cdot 4}{1 \cdot 2} \left(\frac{1}{3} X \right)^3 (2)^2 = 10 \left(\frac{1}{27} \right) X^3 (4) = \frac{40}{27} X^3$$

$$\begin{aligned}
 15. \quad \left(X - \frac{3}{5} \right)^3 &= 1X^3 \left(-\frac{3}{5} \right)^0 + 3X^2 \left(-\frac{3}{5} \right)^1 + \\
 3X^1 \left(-\frac{3}{5} \right)^2 + 1X^0 \left(-\frac{3}{5} \right)^3 &= \\
 X^3 - \frac{9}{5} X^2 + \frac{27}{25} X - \frac{27}{125}
 \end{aligned}$$

$$16. \quad (2X+1)$$

$$17. \quad \frac{10+i}{5i} = \frac{(10+i)(i)}{5i(i)} = \frac{10i+i^2}{5i^2} = \frac{10i-1}{-5}$$

$$\begin{aligned}
 18. \quad \frac{10}{5-\sqrt{8}} &= \frac{10(5+\sqrt{8})}{(5-\sqrt{8})(5+\sqrt{8})} = \frac{50+10\sqrt{8}}{25-8} = \\
 \frac{50+10\sqrt{4}\sqrt{2}}{17} &= \frac{50+10(2)\sqrt{2}}{17} = \frac{50+20\sqrt{2}}{17}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{2+3\sqrt{6}}{1-\sqrt{6}} &= \frac{(2+3\sqrt{6})(1+\sqrt{6})}{(1-\sqrt{6})(1+\sqrt{6})} = \\
 \frac{2+2\sqrt{6}+3\sqrt{6}+3(6)}{1-6} &= \frac{2+5\sqrt{6}+18}{-5} = \\
 \frac{20+5\sqrt{6}}{-5} &= \frac{4+\sqrt{6}}{-1} = -4-\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{6-\sqrt{2}}{10\sqrt{3}-8} &= \frac{(6-\sqrt{2})(10\sqrt{3}+8)}{(10\sqrt{3}-8)(10\sqrt{3}+8)} = \\
 \frac{60\sqrt{3}+48-10\sqrt{6}-8\sqrt{2}}{100(3)-64} &= \\
 \frac{2(30\sqrt{3}+24-5\sqrt{6}-4\sqrt{2})}{300-64} &= \\
 \frac{2(30\sqrt{3}+24-5\sqrt{6}-4\sqrt{2})}{236} &= \\
 \frac{30\sqrt{3}+24-5\sqrt{6}-4\sqrt{2}}{118}
 \end{aligned}$$

Lesson Practice 13A

$$\begin{aligned}
 1. \quad (6)^2 - 4(1)(9) &= 0 \\
 \text{real, rational, equal (double root)}
 \end{aligned}$$

$$\begin{aligned}
 X^2 + 6X + 9 &= 0 \\
 (X+3)(X+3) &= 0 \\
 X+3 &= 0 \\
 X &= -3
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (7)^2 - 4(2)(3) &= 25 \\
 \text{real, rational, unequal}
 \end{aligned}$$

$$\begin{aligned}
 2X^2 + 7X + 3 &= 0 \\
 (2X+1)(X+3) &= 0 \\
 2X+1 &= 0 & X+3 &= 0 \\
 2X &= -1 & X &= -3 \\
 X &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (3)^2 - 4(-2)(6) &= 57 \\
 \text{real, irrational, unequal}
 \end{aligned}$$

$$X = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)(6)}}{2(-2)} = \frac{-3 \pm \sqrt{57}}{-4}$$

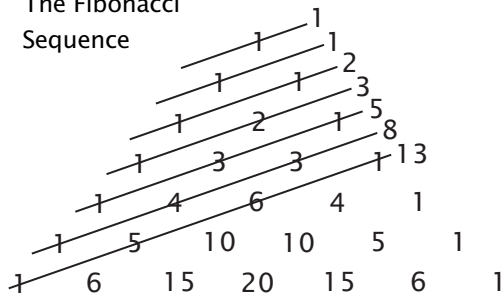
$$\begin{aligned}
 4. \quad (-2)^2 - 4(3)(5) &= -56 \\
 \text{imaginary}
 \end{aligned}$$

$$\begin{aligned}
 X &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)} = \\
 \frac{2 \pm 2i\sqrt{14}}{6} &= \frac{1 \pm i\sqrt{14}}{3}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad 7X^2 - 3X &= 20 \\
 7X^2 - 3X - 20 &= 0 \\
 (-3)^2 - 4(7)(-20) &= 569 \\
 \text{real, irrational, unequal}
 \end{aligned}$$

$$X = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(7)(-20)}}{2(7)} = \frac{3 \pm \sqrt{569}}{14}$$

5. The Fibonacci Sequence



6. $4 \times 3 \times 2 \times 1 = 24$

7. $5 \times 4 \times 3 \times 2 \times 1 = 120$

8. $\frac{9!}{7!} = \frac{9 \times 8 \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}} = 9 \times 8 = 72$

9. $\frac{6!}{3!3!} = \frac{6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{3 \times 2 \times 1 \times \cancel{3} \times \cancel{2} \times \cancel{1}} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 5 \times 4 = 20$

10. $\frac{201!}{200!} = \frac{201 \times \cancel{200!}}{\cancel{200!}} = 201$

Honours Lesson 10

- A, B, C
A, C, B
B, A, C
B, C, A
C, A, B
C, B, A
6 ways

2. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

3. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

4. ${}^9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \times 8 \times 7 \times 6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}} = 9 \times 8 \times 7 \times 6 = 3024$

5. ${}^{20}P_5 = \frac{20!}{(20-5)!} = \frac{20!}{15!} = 20 \times 19 \times 18 \times 17 \times 16 = 1\,860\,480$

6. ${}^{21}P_6 = \frac{21!}{(21-6)!} = \frac{21!}{15!} = 21 \times 20 \times 19 \times 18 \times 17 \times 16 = 39\,070\,080$

Honours Lesson 11

- like ilke klie elik
liek ilek klei elki
leik ikle kiel eilk
leki ikel kile eikl
lkie ielk keli ekli
lkei iekl keil ekil

24 ways; yes

- look olko oklo
loko ookl kloo
lkoo oolk kool
olok okol kolo

12 ways; no

3. $P = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}} = 5 \times 4 \times 3 = 60$

4. $P = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}} = 6 \times 5 \times 4 = 120$

5. $P = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}} = 6 \times 5 \times 4 \times 3 = 360$

6. $P = \frac{6!}{3!2!} = \frac{6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1} \times \cancel{2} \times \cancel{1}} = 5 \times 4 \times 3 = 60$

- m, a, and t each appear twice

$P = \frac{11!}{2!2!2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4} \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1} \times \cancel{2} \times \cancel{1} \times \cancel{2} \times \cancel{1}} = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 3 = 4\,989\,600$

8. $P = \frac{20!}{15!3!2!} = \frac{20 \times 19 \times \cancel{18} \times \cancel{17} \times \cancel{16}}{\cancel{3} \times \cancel{2} \times \cancel{1} \times \cancel{2} \times \cancel{1}} = 20 \times 19 \times 3 \times 17 \times 8 = 155\,040$

Honours Lesson 12

- $\binom{6}{5-1} X^{6-5+1} Y^{5-1} = \binom{6}{4} X^2 Y^4$
 $\frac{6!}{2!4!} X^2 Y^4 = \frac{6 \times 5 \times \cancel{4!}}{2 \times \cancel{4!}} = 15 X^2 Y^4$
- $\binom{4}{2-1} A^{4-2+1} 2^{2-1} = \binom{4}{1} A^3 = 4 A^3$

$$\begin{aligned}
 3. \quad & \binom{5}{3-1} p^{5-3+1} Q^{3-1} = \binom{5}{2} p^3 Q^2 \\
 & \frac{5!}{3!2!} p^3 Q^2 = \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} = 10 p^3 Q^2 \\
 4. \quad & \binom{7}{4-1} (2X)^{7-4+1} (-1^4-1) = \binom{7}{3} (2X)^4 (-1^3) = \\
 & \frac{7!}{4!3!} (-16X^4) = \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} (-16X^4) \\
 & = (35)(-16X^4) = -560X^4
 \end{aligned}$$

Honours Lesson 13

$$\begin{aligned}
 1. \quad & \text{Area} = X(20 - 2X) \\
 & = 20X - 2X^2 \\
 & 48 = 20X - 2X^2 \\
 & 24 = 10X - X^2 \\
 & X^2 - 10X + 24 = 0 \\
 & (X - 6)(X - 4) = 0 \\
 & X = 6, 4 \\
 & \text{If } X = 6 \text{ metres, then the} \\
 & \text{long side would be:} \\
 & 20 - 2(6) = 20 - 12 = 8 \text{ m} \\
 & \text{If } X = 4 \text{ metres, then the} \\
 & \text{long side would be:} \\
 & 20 - 2(4) = 20 - 8 = 12 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \text{Area} = X \left(\frac{160 - 3X}{2} \right) \\
 & = \frac{160X - 3X^2}{2} \\
 1000 & = \frac{160X - 3X^2}{2} \\
 2000 & = 160X - 3X^2 \\
 3X^2 - 160X + 2000 & = 0 \\
 (X - 20)(3X - 100) & = 0 \\
 X & = 20, 33\frac{1}{3} \\
 \text{If } X & = 20, \text{ then the other side would be:} \\
 (160 - 3(20)) \div 2 & = \\
 (160 - 60) \div 2 & = \\
 100 \div 2 & = 50 \text{ m} \\
 20 \times 50 & = 1000 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{If } X & = 33\frac{1}{3}, \text{ the other side} \\
 \text{would be } & \left(160 - 3 \left(33\frac{1}{3} \right) \right) \div 2 = \\
 (160 - 100) \div 2 & = 60 \div 2 = 30 \text{ m} \\
 30 \times 33\frac{1}{3} & = 1000 \text{ m}^2
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \text{Area} = \frac{X(X+2)}{2} \\
 24 & = \frac{X^2 + 2X}{2} \\
 48 & = X^2 + 2X \\
 0 & = X^2 + 2X - 48 \\
 (X+8)(X-6) & = 0 \\
 X & = -8, 6 \\
 X = -8 & \text{ makes no sense.} \\
 \text{If } X & = 6, \text{ then the height is: } (6) + 2 = 8 \\
 \frac{1}{2}(6)(8) & = 24 \text{ cm}^2 \\
 4. \quad & (X)(X+4) = 192 \\
 X^2 + 4X & = 192 \\
 X^2 + 4X - 192 & = 0 \\
 (X+16)(X-12) & = 0 \\
 X & = -16, 12 \\
 X = -16 & \text{ makes no sense.} \\
 \text{If } X & = 12, \text{ then the length is:} \\
 (12) + 4 & = 16 \\
 (12)(16) & = 192 \text{ cm}^2
 \end{aligned}$$

Honours Lesson 14

- Done
- $145\,000 \times 0,26 = R37\,700$ increase
 $145\,000 + 37\,700 = R182\,700$ now
- $0,25 \times 150 =$
 $R37,50$ amount of decrease
 $150 - 37,50 = R112,50$ new price
- $28,5 \times 0,29 =$
 $8,265$ more tonnes per hectare
 $R200 \times 8,265 = R1653$
more per hectare in sales
 $R1653,00 - R75,00 = R1578$
benefit per hectare
- $R1578 \times 150 =$
 $R236\,700$ more than without fertiliser
 $R200 \times 150 \times 28,5 = R855\,000$
 $R236\,700 = WP \times R855\,000$
 $WP = 0,0278$ or 28% (rounded)