

Lesson 12 Quadratic Formula

A quadratic is an equation that has an unknown or variable raised to the second power, as in Y^2 or A^2 . In factoring and in completing the square, we have been dealing exclusively with quadratic equations. So far we can find the solution to a quadratic equation by factoring it, or if this fails, by completing the square. In this lesson we are going to discover a formula to solve all quadratics, by completing the square with variables. If you've mastered the previous lesson, try solving the following equation by completing the square, then compare your solution with mine.

$$AX^2 + BX + C = 0$$

Divide by the coefficient of X^2 .

$$\frac{AX^2}{A} + \frac{BX}{A} + \frac{C}{A} = 0$$

$$X^2 + \frac{BX}{A} + \frac{C}{A} = 0$$

Add the inverse of the third term to both sides.

$$X^2 + \frac{BX}{A} = -\frac{C}{A}$$

Take $1/2$ of the coefficient of the middle term, square it, and add it to both sides.

$$X^2 + \frac{BX}{A} + \left(\frac{B}{2A}\right)^2 = -\frac{C}{A} + \left(\frac{B}{2A}\right)^2$$

Factor the left side.

$$\left(X + \frac{B}{2A}\right)^2 = -\frac{C}{A} + \frac{B^2}{4A^2}$$

Combine terms on the right.

$$\left(X + \frac{B}{2A}\right)^2 = -\frac{4AC}{4A^2} + \frac{B^2}{4A^2}$$

Take the square root of both sides.

$$\sqrt{\left(X + \frac{B}{2A}\right)^2} = \pm \sqrt{-\frac{4AC}{4A^2} + \frac{B^2}{4A^2}}$$

$$X + \frac{B}{2A} = \pm \sqrt{\frac{-4AC + B^2}{4A^2}} = \pm \frac{\sqrt{-4AC + B^2}}{2A}$$

Subtract $B/2A$ from both sides, and combine.

$$X = -\frac{B}{2A} \pm \frac{\sqrt{B^2 - 4AC}}{2A}$$

The quadratic formula!

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Example 1

Let's try one that we know the answer to by factoring, and "plug in" the values for A, B, & C. Remember, to find A, B, & C, you have to be in the form $AX^2 + BX + C = 0$.

$$X^2 + 5X + 6 = 0$$

$$A = 1, B = 5, \text{ \& } C = 6$$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$X = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2}$$

$$X = \frac{-5 \pm 1}{2} = \frac{-4}{2} \text{ or } \frac{-6}{2} = -2 \text{ or } -3$$

We can also solve $X^2 + 5X + 6 = 0$ by factoring.

$$\begin{aligned} X^2 + 5X + 6 &= 0 \\ (X+2)(X+3) &= 0 \\ X+2 &= 0 & X+3 &= 0 \\ X &= -2 & X &= -3 \end{aligned}$$

For this problem it would have much easier to solve by factoring. Try factoring first and if it doesn't work, then use the quadratic formula. Here is another problem to try.

Example 2 Find the factors of $2X^2 = -7X - 4$

To find A, B, & C, you have to be in the form $AX^2 + BX + C = 0$.

$$2X^2 + 7X + 4 = 0$$

$$A = 2, B = 7, \text{ \& } C = 4$$

$$X = \frac{-7 + \sqrt{17}}{4} \text{ or } X = \frac{-7 - \sqrt{17}}{4}$$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$

$$X = \frac{-7 \pm \sqrt{49 - 32}}{4} = \frac{-7 \pm \sqrt{17}}{4}$$

$$X = \frac{-7 \pm \sqrt{17}}{4}$$

Practice Problems Solve for X. First try factoring, then use the quadratic formula if necessary.

1) $X^2 - 25 = 0$

5) $4A^2 - 36 = 0$

9) $\frac{5}{X+3} + \frac{2}{X-3} = 5 \quad (X \neq \pm 3)$

2) $X^2 + 5 = -3X$

6) $X^2 - 18X = -81$

10) $4X^2 = 9$

3) $2X^2 + 7X + 6 = 0$

7) $7X^2 = -2X + 1$

11) $3Q^2 = -4Q - 2$

4) $3X^2 + X - 4 = 0$

8) $2X^2 + 2X - 5 = 0$

12) $4X^2 + 20X = -25$

Solutions

1) $(X+5)(X-5) = 0$

$$X+5=0 \quad X-5=0$$

$$X = 5, -5$$

5) $(2A-6)(2A+6) = 0$

$$2A-6=0 \quad 2A+6=0$$

$$X = 3, -3$$

9) $(X^2 - 9) \left(\frac{5}{X+3} + \frac{2}{X-3} \right) = 5$

$$5(X-3) + 2(X+3) = 5(X^2 - 9)$$

$$7X - 9 = 5X^2 - 45$$

$$5X^2 - 7X - 36 = 0$$

$$X = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 5 \cdot -36}}{2 \cdot 5}$$

$$X = \frac{7 \pm \sqrt{769}}{10}$$

$$X = \frac{7 + \sqrt{769}}{10} \text{ or } X = \frac{7 - \sqrt{769}}{10}$$

2) $X = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$

$$X = \frac{-3 \pm \sqrt{-11}}{2}$$

$$X = \frac{-3 + \sqrt{-11}}{2} \text{ or } X = \frac{-3 - \sqrt{-11}}{2}$$

6) $(X-9)(X-9) = 0$

$$X-9=0 \quad X-9=0$$

$$X = 9$$

7) $X = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 7 \cdot -1}}{2 \cdot 7}$

$$X = \frac{-2 \pm 4\sqrt{2}}{14}$$

$$X = \frac{-1 + 2\sqrt{2}}{7} \text{ or } X = \frac{-1 - 2\sqrt{2}}{7}$$

3) $(2X+3)(X+2) = 0$

$$2X+3=0 \quad X+2=0$$

$$X = -3/2, -2$$

10) $(2X-3)(2X+3) = 0$

$$2X-3=0 \quad 2X+3=0$$

$$X = 3/2, -3/2$$

4) $(3X+4)(X-1) = 0$

$$3X+4=0 \quad X-1=0$$

$$X = -4/3, 1$$

8) $X = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2}$

$$X = \frac{-2 \pm 2\sqrt{11}}{4}$$

$$X = \frac{-1 + \sqrt{11}}{2} \text{ or } X = \frac{-1 - \sqrt{11}}{2}$$

11) $X = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3}$

$$X = \frac{-2 + i\sqrt{2}}{3} \text{ or } X = \frac{-2 - i\sqrt{2}}{3}$$

12) $(2X+5)(2X+5) = 0$

$$2X+5=0 \quad 2X+5=0$$

$$X = -5/2$$