Honors Solutions

Honors Lesson 1

1. A. Multiplying by $\frac{1}{2}$:

$$1 \div \frac{3}{2} \times \frac{3}{4} = 1 \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

$$1 \times \frac{1}{2} = \frac{1}{2}$$

2. Substitute 3 for $(r - s)$:

$$3(3) + \frac{(3)}{18} - (3)^2 - 3 =$$

$$9 + \frac{1}{6} - 9 - 3 =$$

$$\frac{1}{6} - 3 = -2\frac{5}{6}$$

3. Since it is a square, we know all 4 sides are equal, therefore:

$X + 9 = 4X$

$9 = 3X$

$3 = X$

4. $A = (X + 9)(4X)$ square units

using $X = 3$ from #3

$A = (3 + 9)(4 \cdot 3)$

$A = (12)(12) = 144$ square units

5. $(X + 9)(4X) = 144$

$4X^2 + 36X = 144$

$4X^2 + 36X - 144 = 0$

$X^2 + 9X - 36 = 0$

$(X - 3)(X + 12) = 0$

$X = 3$ same as #3

$X = -12$ This solution does not make sense. We say that it is invalid.

6. 4 : 5

$$\frac{4}{5} = \frac{8}{C}$$

$4C = 40$

$C = 10$

7. $\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

8. $180 - 35 = 145°$

9. $\frac{X^4Y^2 + X^2Y}{X^2Y} = X^2Y + 1$

10. Plug in values for X and Y:

$(2)^2(3) + 1 = (4)(3) + 1 = 12 + 1 = 13$ one side

$(2)^2(3) = (4)(3) = 12$ other side

Area $= 13 \times 12 = 156$ square units

Honors Lesson 2

1.
$$\frac{t}{8} + \frac{t}{12} = 1$$

$$24\left(\frac{t}{8} + \frac{t}{12}\right) = 24$$

$$3t + 2t = 24$$

$$5t = 24$$

$t = 4\frac{4}{5}$ hours or 4 hours and 48 minutes

2.
$$\frac{t}{30} + \frac{t}{45} = 1$$

$$3t + 2t = 90 \quad \begin{pmatrix}\text{multiplied both}\\ \text{sides by 60}\end{pmatrix}$$

$$5t = 90$$

$t = 18$ minutes

3.
$$\frac{t}{20} + \frac{t}{10} + \frac{t}{12} = 1$$

$$3t + 6t + 5t = 60 \quad \begin{pmatrix}\text{multiplied both}\\ \text{sides by 60}\end{pmatrix}$$

$$14t = 60$$

$t = 4\frac{2}{7}$ days

4. Subtract this time, since the faucet and the drain are working against each other:

$$\frac{t}{15} - \frac{t}{20} = 1$$

$$4t - 3t = 60 \quad \begin{pmatrix}\text{multiplied both}\\ \text{sides by 60}\end{pmatrix}$$

$t = 60$ minutes or 1 hour

Honors Lesson 3

1. rate of work × time worked = portion of job done

1/6	2 hours	1/3
1/10	6 2/3 hours	2/3

The rates have already been filled in. We are given the amount of time that the gardener worked, so we fill that in, and then figure out how much of the job he completed. If 1/3 of the job is done, then 2/3 of the job is left. Fill in that amount, and then figure the time worked by the helper by using the values and solving for time.

$RT = J$

$\left(\frac{1}{6}\right)(2) = J$

$\frac{1}{3} = J$

$RT = J$

$\left(\frac{1}{10}\right)(T) = \frac{2}{3}$

$T = \frac{20}{3}$

$T = 6\frac{2}{3}$ or 6 hours 40 minutes

2. 5

3. 1

4. 4

5. 2

6. 2

7. 2

8. 3

9. 4

10. 2.45×10^{8} ft; 3 significant digits

11. 9×10^{-5} m; 1 significant digit

12. 1.304×10^{3} tons; 4 significant digits

13. 1.50×10^{0} g; 3 significant digits

Honors Lesson 4

1.

rate of work	×	time worked	= portion of job done
$\frac{1}{12}$		3 hours	$\frac{1}{4}$
$\frac{1}{20}$		15 hours	$\frac{3}{4}$

The mason works at the rate of 1/12 of the job per hour, and he worked for 3 hours. We also know his helper worked for a total of 15 hours. Using the formula, we find that the mason did 1/4 of the job. His helper, therefore, did 3/4 of the job. Use the formula again to find out the helper's rate:

$RT = J$

$R(15) = \frac{3}{4}$

$R = \frac{3}{60}$ or $\frac{1}{20}$

Working alone, the helper would have taken 20 hours to do the job.

2. $250 + 12.5 = 262.5$ ft; round to 260 ft

3. $.5 - .361 = .139$ in; round to .1 in

4. $(5.8 \times 0^4) + (1.2 \times 10^{-2}) = 58{,}000 + .012 = 58{,}000.012$ m

round to 58,000 m or 5.8×10^4 m

5. $650{,}000 - 3{,}400 = 646{,}600$ g; round to 650,000 or 6.5×10^5 g

6. 151 x 6 = 906 ft^2; round to 900 ft^2

7. $.0025 \div .10 = .025$ in; two significant digits

8. $(2.8 \times 10^2) \times (1.04 \times 10^2) = (2.912 \times 10^4)$ m^2; round to (2.9×10^4) m^2

9. $(3.6 \times 10^8) \div (1.2 \times 10^4) = 3.0 \times 10^4$ km; two significant digits

10. Area $= 19.1 \times 6 = 114.6$; round to 100 m^2 (one significant digit)

Perimeter $= 19.1 + 6 + 19.1 + 6 = 50.2$ m; round to 50 m

Honors Lesson 5

1. $d = rt$
   $d = (3)(40)$
   $d = 120$ miles
2. $d = (6)(40) = 240$ miles
   240 is twice 120, so increased by a factor of 2
3. $d = (5)(60) = 300$ miles
   $d = (10)(60) = 600$ miles
   600 is twice 300, so increased by a factor of 2
4. It will double, or increase by a factor of 2.
5. $5^2 = 25$
6. $10^2 = 100$
   100 is 4 times 25, so a factor of 4
7. $4^2 = 16$
   $8^2 = 64$
   64 is 4 times 16, so a factor of 4
8. The value of X should increase by a factor of 4.
9. $L = 12 \div 2$
   $L = 6$
10. $L = 12 \div 4$
    $L = 3$
    The length decreases as the width increases.
    Doubling the length decreases the width by a factor of 2.
11. The value of X should decrease by a factor of 2.

Honors Lesson 6

1. X
2. Use the pythagorean theorem:
   $A^2 + B^2 = C^2$
   substitute X for A, and 2X for B:
   $X^2 + (2X)^2 = C^2$
   $5X^2 = C^2$
   $\sqrt{5X^2} = C$
   $X\sqrt{5} = C$
3. $X\sqrt{5} + X$
4. $X\sqrt{5} - X$
5. $\frac{X\sqrt{5} + X}{2X}$
6. $\frac{\sqrt{5} + 1}{2}$
7. 1.618
8. answers will vary
9. $1 \div 1.618 = .618$
10. $\frac{5}{8}$ is close $(.625)$, but you may have come up with something closer.

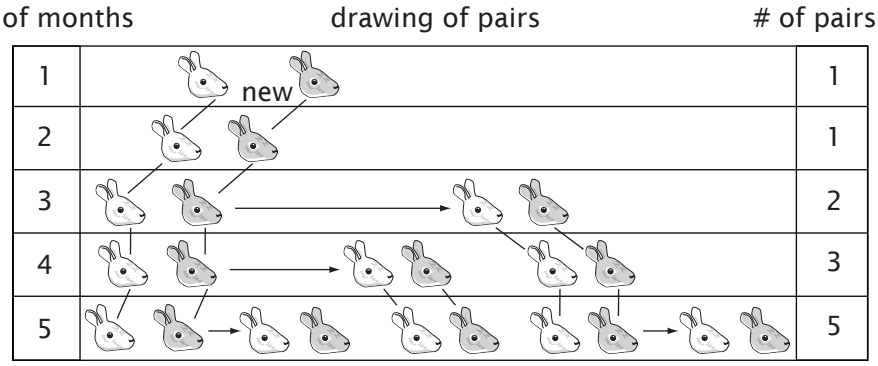
Honors Lesson 7

1. $A^{\frac{X}{Y}} = \left(\sqrt[Y]{A}\right)^X$
2. Q times itself R times
3. $\left(X^{\frac{a}{b}}\right)^{\frac{b}{a}} = X$
4. $\left(Y^{\frac{a}{b}}\right)^{\frac{c}{d}} = \left(\sqrt[bd]{Y}\right)^{ac}$
5. $\left(Y^F \cdot Y^G\right)^{\frac{1}{H}} = \left(\sqrt[H]{Y}\right)^{F+G}$
6. $\left(X^F \cdot Y^F\right)^G = X^{FG}Y^{FG}$
7. $\left(M^{\frac{x}{z}} \cdot M^{\frac{y}{z}}\right)^{\frac{z}{y}} = \left(M^{\frac{x+y}{z}}\right)^{\frac{z}{y}} = M^{\frac{x+y}{y}}$

8. $\left[\left(x^a\right)^b \cdot x^b\right]^{\frac{1}{c}} = \sqrt[c]{\left(x^{ab+b}\right)}$
9. $\left(p^a + p^a\right)^{\frac{a}{b}} = \left(\sqrt[b]{2p^a}\right)^a$
10. $\left(x^E \div x^F\right)^H = \left(x^{E-F}\right)^H$

Honors Lesson 8

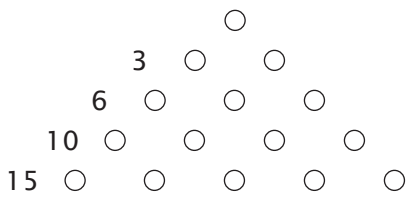
1. 0
2. a negative number that is not a fraction or decimal, for example: –6
3. any fraction, for example: $\frac{3}{5}$
4. π, $\sqrt{2}$, $\sqrt{3}$, etc.
5. see chart below
6. Each number in the series is the sum of the previous two numbers
7. 8, 13, 21



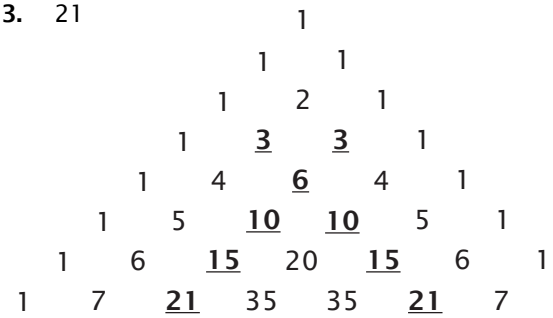
Honors Lesson 9

1. Row 0 : 1
   Row 1: 2
   Row 2: 4
   Row 3: 8
   Row 4: 16
   Row 5: 32
   Row 6: 64
   The sum of each row is twice the previous row.

2.

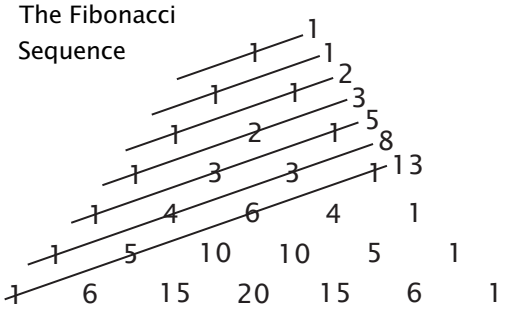


3. 21



4. $3+6=9$; $6+10=16$; $10+15=25$;
$15+21=36$ They are all perfect squares.

5. The Fibonacci Sequence



6. $4 \times 3 \times 2 \times 1 = 24$

7. $5 \times 4 \times 3 \times 2 \times 1 = 120$

8. $\frac{9!}{7!} = \frac{9 \times 8 \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$
$= 9 \times 8 = 72$

9. $\frac{6!}{3!3!} = \frac{6 \times 5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{3 \times 2 \times 1 \times \cancel{3} \times \cancel{2} \times \cancel{1}}$
$\frac{\cancel{6} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1} = 5 \times 4 = 20$

10. $\frac{201!}{200!} = \frac{201 \times \cancel{200!}}{\cancel{200!}} = 201$

Honors Lesson 10

1. A, B, C
A, C, B
B, A, C
B, C, A
C, A, B
C, B, A
6 ways

2. $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

3. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

4. ${}_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} =$
$\frac{9 \times 8 \times 7 \times 6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$
$= 9 \times 8 \times 7 \times 6 = 3{,}024$

5. ${}_{20}P_5 = \frac{20!}{(20-5)!} = \frac{20!}{15!} =$
$20 \times 19 \times 18 \times 17 \times 16 =$
1,860,480

6. ${}_{21}P_6 = \frac{21!}{(21-6)!} = \frac{21!}{15!} =$
$21 \times 20 \times 19 \times 18 \times 17 \times 16 =$
39,070,080

Honors Lesson 11

1.

like	ilke	klie	elik
liek	ilek	klei	elki
leik	ikle	kiel	eilk
leki	ikel	kile	eikl
lkie	ielk	keli	ekli
lkei	iekl	keil	ekil

24 ways; yes

2.

look	olko	oklo
loko	ookl	kloo
lkoo	oolk	kool
olok	okol	kolo

12 ways; no

3. $P = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} =$
$5 \times 4 \times 3 = 60$

4. $P = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1}} =$

$6 \times 5 \times 4 = 120$

5. $P = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} =$

$6 \times 5 \times 4 \times 3 = 360$

6. $P = \frac{6!}{3!2!} =$

$\frac{\overset{3}{\cancel{6}} \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1} \times \cancel{2 \times 1}} = 5 \times 4 \times 3 = 60$

7. m, a, and t each appear twice

$P = \frac{11!}{2!2!2!} =$

$\frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4} \times 3 \times \cancel{2 \times 1}}{\cancel{2} \times 1 \times \cancel{2} \times 1 \times \cancel{2 \times 1}} =$

$11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 3 = 4{,}989{,}600$

8. $P = \frac{20!}{15!3!2!} = \frac{20 \times 19 \times \overset{3}{\cancel{18}} \times 17 \times \overset{8}{\cancel{16}}}{\cancel{3 \times 2} \times 1 \times \cancel{2} \times 1} =$

$20 \times 19 \times 3 \times 17 \times 8 = 155{,}040$

Honors Lesson 12

1. $\binom{6}{5-1}X^{6-5+1}Y^{5-1} = \binom{6}{4}X^2Y^4$

$\frac{6!}{2!4!}X^2Y^4 = \frac{6 \times 5 \times \cancel{4!}}{2 \times \cancel{4!}} = 15X^2Y^4$

2. $\binom{4}{2-1}A^{4-2+1}2^{2-1} = \binom{4}{1}A^3 2 = 8A^3$

3. $\binom{5}{3-1}P^{5-3+1}Q^{3-1} = \binom{5}{2}P^3Q^2$

$\frac{5!}{3!2!}P^3Q^2 = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2 \times 1} = 10P^3Q^2$

4. $\binom{7}{4-1}(2X)^{7-4+1}\left(-1^{4-1}\right) = \binom{7}{3}(2X)^4\left(-1^3\right) =$

$\frac{7!}{4!3!}\left(-16X^4\right) = \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times \cancel{2} \times 1}\left(-16X^4\right)$

$= (35)\left(-16X^4\right) = -560X^4$

Honors Lesson 13

1. $\text{Area} = X(20 - 2X)$

$= 20X - 2X^2$

$48 = 20X - 2X^2$

$24 = 10X - X^2$

$X^2 - 10X + 24 = 0$

$(X - 6)(X - 4) = 0$

$X = 6,\ 4$

If X = 6 feet, then the long side would be:

$20 - 2(6) = 20 - 12 = 8$ ft

If X = 4 feet, then the long side would be:

$20 - 2(4) = 20 - 8 = 12$ ft

2. $\text{Area} = X\left(\frac{160 - 3X}{2}\right)$

$= \frac{160X - 3X^2}{2}$

$1{,}000 = \frac{160X - 3X^2}{2}$

$2000 = 160X - 3X^2$

$3X^2 - 160X + 2{,}000 = 0$

$(X - 20)(3X - 100) = 0$

$X = 20,\ 33\frac{1}{3}$

If X = 20, then the other side would be:

$(160 - 3(20)) \div 2 =$

$(160 - 60) \div 2 =$

$100 \div 2 = 50$ ft

$20 \times 50 = 1{,}000$ ft^2

If $X = 33\frac{1}{3}$, the other side would be:

$\left(160 - 3\left(33\frac{1}{3}\right)\right) \div 2 =$

$(160 - 100) \div 2 =$

$60 \div 2 = 30$ ft

$30 \times 33\frac{1}{3} = 1000$ ft^2

3. $\text{Area} = \frac{X(X+2)}{2}$

$$24 = \frac{X^2+2X}{2}$$
$$48 = X^2+2X$$
$$0 = X^2+2X-48$$
$$(X+8)(X-6) = 0$$
$$X = -8,\ 6$$

$X = -8$ makes no sense

If $X = 6$, then the height is:

$$(6)+2 = 8$$
$$\frac{1}{2}(6)(8) = 24 \text{ in}^2$$

4. $$(X)(X+4) = 192$$
$$X^2+4X = 192$$
$$X^2+4X-192 = 0$$
$$(X+16)(X-12) = 0$$
$$X = -16,\ 12$$

$X = -16$ makes no sense

If $X = 12$, then the length is:

$$(12)+4 = 16$$
$$(12)(16) = 192 \text{ in}^2$$

Honors Lesson 14

1. Done

2. $145{,}000 \times .26 = \$37{,}700$ increase
$145{,}000 + 37{,}700 = \$182{,}700$ now

3. $.25 \times 150 = \$37.50$ amount of decrease
$150 - 37.50 = \$112.50$ new price

4. $28.5 \times .29 = 8.265$ more bushels per acre
$4.15 \times 8.265 = \$34.30$ more per acre in sales
$34.30 - 25.00 = \$9.30$ benefit per acre

5. $9.30 \times 150 =$
$\$1{,}395.00$ more than without fertilizer
$4.15 \times 150 \times 28.5 = \$17{,}741.25$
$1{,}395 = \text{WP} \times 17{,}741.25$
WP = .079 or 7.9% (rounded)

6. $29{,}352 - 20{,}578 = 8{,}774$ increase
$8{,}774 = \text{WP} \times 20{,}578$
WP = .426 or 42.6% increase (rounded)
$.426 \times 29{,}352 =$
$\$12{,}503.95$ increase next year
$29{,}352 + 12{,}503.95 =$
$\$41{,}855.95$ in sales next year
if there is the same percentage increase

7. $4.7 - 4.1 = .6$ gallons saved
$.6 = \text{WP} \times 4.7$
WP = .128 or 12.8% (rounded)

8. .6 gallons saved per hundred miles driven, so
$.6 \times 6 = 3.6$ gallons saved
$3.6 \times 1.98 = \$7.13$ saved (rounded)

9. $20{,}567 \times 4.00 = 82{,}268$

10. $82{,}268 - 20{,}567 = 61{,}701$ increase
$61{,}701 = \text{WP} \times 20{,}567$
WP = 3 or 300%

Honors Lesson 15

1. $\frac{E}{h} = f$

2. $PA = F$
$A = \frac{F}{P}$

3. $$P = 2L+2W$$
$$P-2L = 2W$$
$$\frac{P-2L}{2} = W$$

4. $kT = PV$
$P = \frac{kT}{V}$

5. $$N = \frac{a+b}{2}$$
$$2N = a+b$$
$$2N-b = a$$

6. $$M = \frac{a+b}{c+d}$$
$$M(c+d) = a+b$$
$$c+d = \frac{a+b}{M}$$
$$c = \frac{a+b}{M} - d$$

7. It will increase.

$t = 2,\ r = 40 : d = (2)(40) = 80$

$t = 2,\ r = 60 : d = (2)(60) = 120$

8. It will decrease.

$t = 2,\ r = 40 : d = (2)(40) = 80$

$t = 1,\ r = 40 : d = (1)(40) = 40$

9. R will increase as E increases.

10. R will decrease as i increases.

Honors Lesson 16

1. The smaller gear will move faster.

2.
$$RN = rn$$
$$120(12) = r(6)$$
$$1{,}440 = 6r$$
$$r = 240 \text{ rpm}$$

3.
$$RN = rn$$
$$\frac{RN}{r} = n \text{ Divide both sides by r.}$$
$$\frac{R}{r} = \frac{n}{N} \text{ Divide both sides by N.}$$

4. $N = \frac{rn}{R}$ $R = \frac{rn}{N}$

$n = \frac{RN}{r}$ $r = \frac{RN}{n}$

5.
$$r = \frac{RN}{n}$$
$$r = \frac{300(40)}{30}$$
$$r = \frac{12{,}000}{30}$$
$$r = 400 \text{ rpm}$$

6.
$$N = \frac{rn}{R}$$
$$N = \frac{150(55)}{50}$$
$$N = \frac{8{,}250}{50}$$
$$N = 165 \text{ teeth}$$

7.
$$R = \frac{rn}{N}$$
$$R = \frac{600(60)}{90}$$
$$R = \frac{36{,}000}{90}$$
$$R = 400 \text{ rpm}$$

8.
$$2{,}000(10) = r(4{,}000)$$
$$20{,}000 = 4{,}000r$$
$$r = 5 \text{ in}$$

Honors Lesson 17

1.
$$X^4 + 3X^2 - 10$$
$$W^2 + 3W - 10$$
$$(W + 5)(W - 2)$$
$$(X^2 + 5)(X^2 - 2)$$

2.
$$X^4 - 8X^2 + 12$$
$$W^2 - 8W + 12$$
$$(W - 2)(W - 6)$$
$$(X^2 - 2)(X^2 - 6)$$

3.
$$X + 3\sqrt{X} + 2$$
$$W^2 + 3W + 2$$
$$(W + 1)(W + 2)$$
$$(\sqrt{X} + 1)(\sqrt{X} + 2)$$

4.
$$\frac{X - 2}{-X^2 + 3X - 2} = \frac{X - 2}{(-1)(X^2 - 3X + 2)} =$$
$$\frac{\cancel{X-2}}{(-1)(X - 1)(\cancel{X-2})} = \frac{1}{-X + 1} \text{ or } \frac{1}{1 - X}$$

5.
$$\frac{3 - X}{X^2 - 9} = \frac{(-1)(\cancel{X-3})}{(X + 3)(\cancel{X-3})} = \frac{-1}{X + 3}$$

6.
$$\frac{X^2 - 4}{2 - X} \cdot \frac{X + 3}{9 - X^2} =$$
$$\frac{(X + 2)(\cancel{X-2})}{(-1)(\cancel{X-2})} \cdot \frac{\cancel{X+3}}{(-1)(X - 3)(\cancel{X+3})} = \frac{X + 2}{X - 3}$$

Honors Lesson 18

1. center rectangle:

$(X+3)\left[(X+1)+2\right] =$

$(X+3)(X+3) = X^2+6X+9$

2 smaller rectangles:

$2\left[(2)(X+1)\right] = 4X+4$

together:

$X^2+6X+9+4X+4 = X^2+10X+13$ units^2

2. $(3)^2+10(3)+13 =$

$9+30+13 = 52$ ft^2

3. lower section:

$(X)(2X+4)(X+3) =$

$\left(2X^2+4X\right)(X+3) =$

$2X^3+4X^2+6X^2+12X = 2X^3+10X^2+12X$

top section:

$(2X)(X+3)\left((2X+4)-2X\right) =$

$\left(2X^2+6X\right)(4) = 8X^2+24X$

together :

$2X^3+10X^2+12X+8X^2+24X =$

$2X^3+18X^2+36X$ units^3

4. $\frac{4}{3}\pi(X+2)^3 = \frac{4}{3}\pi(X^3+6X^2+12X+8)$ or

$\frac{4}{3}\pi X^3+8\pi X^2+16\pi X+\frac{32}{3}\pi$

5. Answers may vary – choose a value for n and raise to the sixth power. For example, $3^6 = 729$. Other possibilities are: 1; 729; 4,096; 15,625

6. $n^{10} = n^5 \times n^5 = \left(n^5\right)^2$

$n^{10} = n^2 \times n^2 \times n^2 \times n^2 \times n^2 = \left(n^2\right)^5$

Honors Lesson 19

1. $\frac{N_P}{N_S} = \frac{E_P}{E_S}$

$\frac{100}{20} = \frac{600}{E_S}$

$100E_S = 12{,}000$

$E_S = 120$ volts

2. $\frac{N_P}{N_S} = \frac{E_P}{E_S}$

$\frac{480}{N_S} = \frac{7{,}200}{240}$

$7{,}200N_S = 480(240)$

$7{,}200N_S = 115{,}200$

$N_S = 16$ turns

3. $\frac{N_P}{N_S} = \frac{E_P}{E_S}$

$\frac{500}{300} = \frac{E_P}{750}$

$300E_P = 500(750)$

$300E_P = 375{,}000$

$E_P = 1{,}250$ volts

Honors Lesson 20

1. $\rho = \frac{m}{V}$

$m = V\rho$

$m = (10)(.009) = .09$

2. $f = \frac{1}{T}$

$T = \frac{1}{f}$

$T = \frac{1}{1.3} = .77$ (rounded)

3. $PE = mgh$

$$h = \frac{PE}{mg}$$

$$h = \frac{1764}{(30)(9.8)} = 6$$

4. $F = \frac{kq_1q_2}{r^2}$

$$r^2 = \frac{kq_1q_2}{F}$$

$$r^2 = \frac{(9.0\times10^{9})(4.0\times10^{-2})(2.0\times10^{-3})}{1.8\times10^{5}}$$

$$r^2 = \frac{72\times10^{4}}{1.8\times10^{5}} = 40\times10^{-1} = 4$$

$$r = 2$$

5. $PV = nRT$

$$V = \frac{nRT}{P}$$

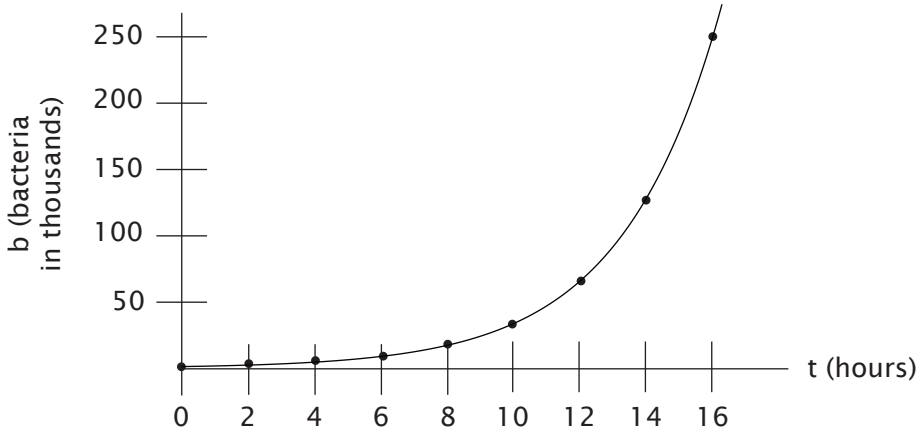
$$V = \frac{(.5)(.0821)(293)}{.95} = 12.66 \text{ (rounded)}$$

Honors Lesson 21

1.

t(hours)	0	2	4	6	8	10	12	14	16
b (bacteria in thousands)	1	2	4	8	16	32	64	128	256

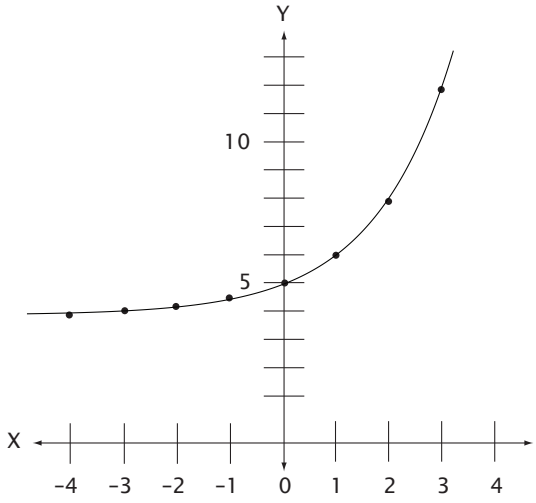
2.



3.

X	Y
0	5
1	6
2	8
3	12
−1	4.5
−2	4.25
−3	4.125

4.



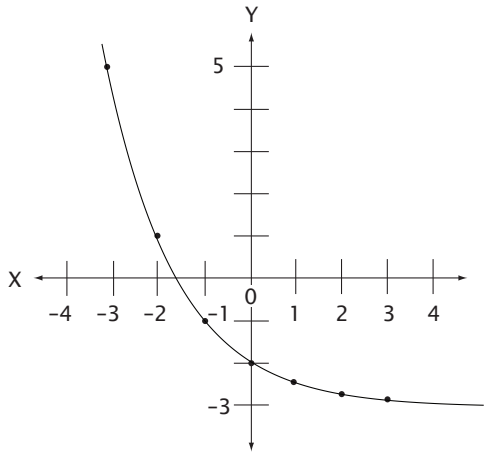
5. Y increases faster and faster.

Honors Lesson 22

1.

X	Y
0	−2
1	−2.5
2	−2.75
3	−2.875
−1	−1
−2	1
−3	5

2.

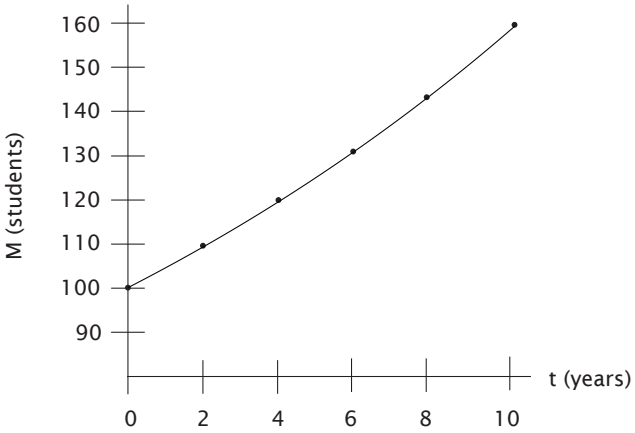


3. $M = A(1.1)^{t/d}$

$M = 100(1.1)^{4/2}$

$M = 100(1.1)^2 = 121$ students

4.



5. Since the number increases by a factor of 1.1 every 2 years, see how many times you have to multiply the original number by 1.1 to reach 133, and then multiply by 2:

$(100)(1.1) = 110$

$(110)(1.1) = 121$

$(121)(1.1) = 133.1$ so 6 years

Honors Lesson 23

1. $YX - YT = YZ$

$X - T = Z$

$X = Z + T$

2. $Q(X + B) = R(X + C)$

$QX + QB = RX + RC$

$QX - RX = RC - QB$

$X(Q - R) = RC - QB$

$$X = \frac{RC - QB}{Q - R}$$

3. $AX - BX - C = CX + X + E$

$AX - BX - CX - X = E + C$

$X(A - B - C - 1) = E + C$

$$X = \frac{E + C}{A - B - C - 1}$$

4. $X(A + B + C) + Y - Z = A$

$X(A + B + C) = A - Y + Z$

$$X = \frac{A - Y + Z}{A + B + C}$$

5. $C(X - Y) + F = CAB - CY + F$

$C(X - Y) = CAB - CY$

$X - Y = AB - Y$

$X = AB$

Your work for the following problems may look different, depending on which equation was substituted into the other. You should have the same final answers.

6. $Y = R + 2X$

$Y = S + X$

Substitute S + X for Y in the first equation:

$S + X = R + 2X$

$S - R = 2X - X$

$S - R = X$

7. $Y = EX$

$Y + EX = Q$

Substitute EX for Y in the second equation:

$EX + EX = Q$

$X(E + E) = Q$

$$X = \frac{Q}{2E}$$

8. $X = Y + A$

$X = BY - B$

Solve first equation for Y: $X - A = Y$

Substitute X – A for Y in the second equation:

$X = B(X - A) - B$

$X = BX - AB - B$

$X - BX = -AB - B$

$X(1 - B) = -AB - B$

$$X = \frac{-AB - B}{1 - B}$$

9. $Y - X = Q$

$Y + RX = T$

Solve the first equation for Y:

$Y = X + Q$

Substitute X + Q for Y in the second equation:

$(X + Q) + RX = T$

$X + RX = T - Q$

$X(1 + R) = T - Q$

$$X = \frac{T - Q}{1 + R}$$

10. $Y - CX = C$

$Y + DX = -D$

Solve the first equation for Y:

$Y = C + CX$

Substitute C + CX for Y in the second equation:

$C + CX + DX = -D$

$CX + DX = -D - C$

$X(C + D) = -D - C$

$$X = \frac{-D - C}{C + D}$$

Honors Lesson 24

1. $AY + BX = C$
$-AY - DX = E$
$(B - D)X = C + E$
$X = \frac{C+E}{B-D}$

2. $X - Y = R$
$AX + Y = T$
$AX + X = R + T$
$X(A + 1) = R + T$
$X = \frac{R+T}{A+1}$
$\frac{R+T}{A+1} - Y = R$
$\frac{R+T}{A+1} - R = Y$

3. $Y - QX = R$
$QY + QX = QT$
$(1 + Q)Y = R + QT$
$Y = \frac{R+QT}{1+Q}$
$Y = \frac{(2)+(3)(6)}{1+(3)}$
$Y = \frac{20}{4} = 5$

Honors Lesson 25

1. minimum

2. maximum

3. minimum

4. $A = LW$
$L = 400 - 2W$
$A = W(400 - 2W) = -2W^2 + 400W$
$A = W^2 - 200W$
$\frac{-b}{2a} = \frac{-(-200)}{2(1)} = 100$
If $W = 100$ then:
$L = 400 - 2(100) = 200$
100×200 encloses the largest area

5. $\frac{-b}{2a} = \frac{-(-2)}{2(.4)} =$
2.5 thousand or 2,500 blouses

6. find maximum of $2X^2 - 60X - 36{,}000$:
$\frac{-b}{2a} = \frac{-(-60)}{2(2)} = 15$
$X = 15$ units of 20,000
$X = 300{,}000$ gallons

7. $\$1.20 + .15 = \$1.35$

8. $3{,}000{,}000 - 300{,}000 =$
2,700,000 gallons sold
\$1.35 x 2,700,000 =
\$3,645,000

Honors Lesson 26

1. $H = 0.2X^2 - 0.5X + 30$
minimum $= \frac{-(-.5)}{2(.2)} = 1.25$
$H = .2(1.25)^2 - .5(1.25) + 30$
$H = 29.6875$
$H = 29.7$ ft (rounded)

2. $P = -0.2X^2 + 50X$
maximum $= \frac{-50}{2(-.2)} = 125$
$P = -.2(125)^2 + 50(125) = \$3{,}125$

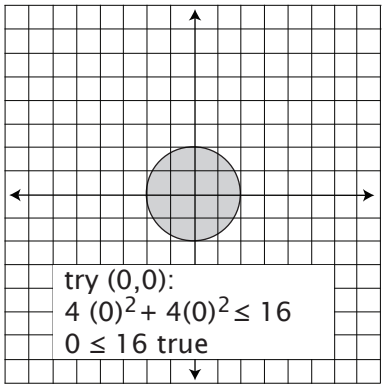
3. $H = -0.002X^2 + .5X + 3$
$\frac{-.5}{2(-.002)} = 125$ ft
The ball will be at its highest point at 125 feet from where it was thrown, so after that, it will be descending.
Find the value of H when X is 200:
$H = -0.002(200)^2 + .5(200) + 3$
$H = -0.002(40{,}000) + 100 + 3$
$H = -80 + 100 + 3 = 23$ ft yes

4. $C = 0.3X^2 - 3X + 12$
$\frac{-(-3)}{2(.3)} = 5$ tons

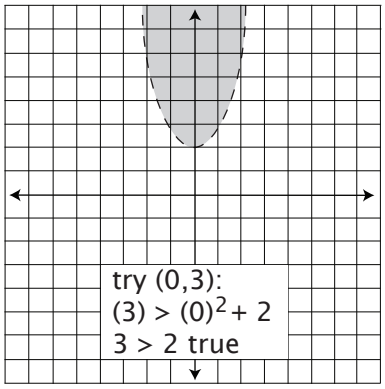
5. $C = 0.3(5)^2 - 3(5) + 12$
$C = .3(25) - 15 + 12$
$C = 7.5 - 15 + 12 = \$4.50$

Honors Lesson 27

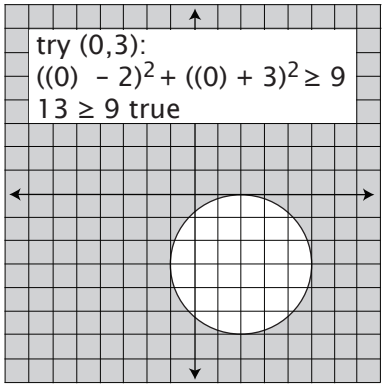
1.



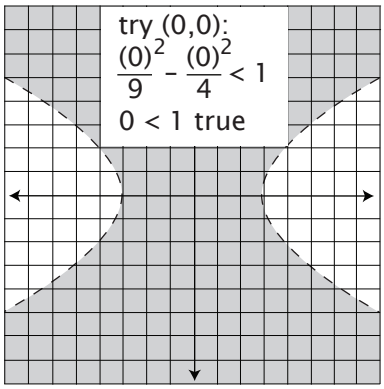
2.



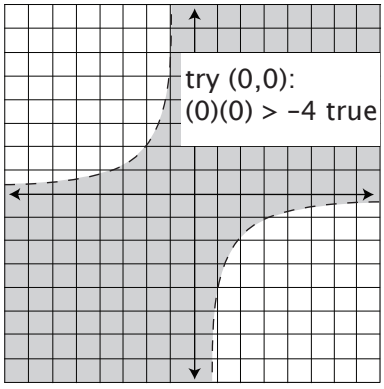
3.



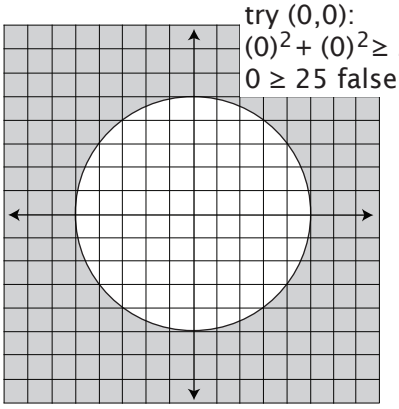
4.



5.



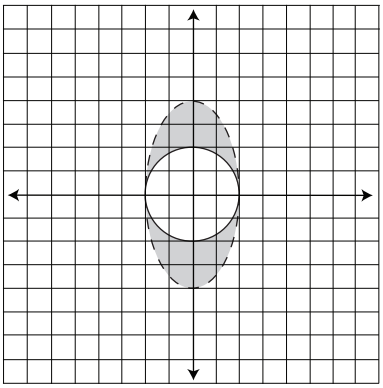
6.



Honors Lesson 28

For clarity, only the final solutions are shaded.

1.



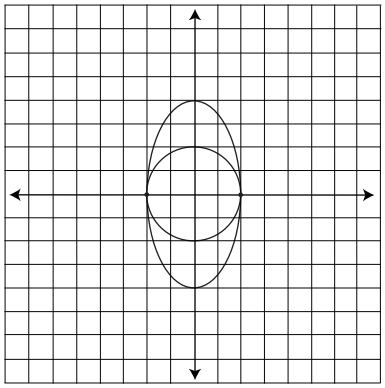
ellipse: $4(0)^2 + (0)^2 < 16$

$0 < 16$ true

circle: $(0)^2 + (0)^2 \geq 4$

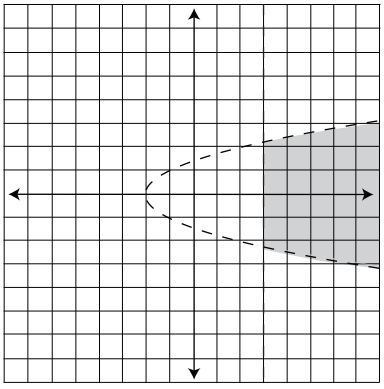
$0 \geq 4$ false

2.



The intersection is in the points $(-2, 0)$ and $(2, 0)$

3.

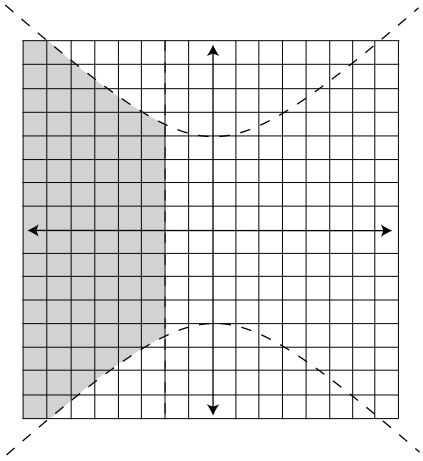


parabola: $(0) > (0)^2 - 2$

$0 > -2$ true

line: $(0) > 3$ false

4.



hyperbola: $(0)^2 - (0)^2 < 16$ true

line: $(0) < -2$ false

For #1, 3, and 4, it is a good idea to check additional points to accurately determine the shaded areas.

Honors Lesson 29

1.

	amount	cost per unit	value
low grade	X lb	2.00	2X
ground round	50 lb	3.75	187.50
mixture	X + 50 lb	2.50	2.50 (X + 50)

$$2X + 187.50 = 2.50(X + 50)$$
$$2X + 187.5 = 2.5X + 125$$
$$187.5 - 125 = 2.5X - 2X$$
$$62.5 = .5X$$
$$X = 125 \text{ lb}$$

2.

	amount	cost per unit	value
metal 1	12 lb	2.00	24.00
metal 2	12 lb	X	12X
mixture	24	1.25	30.00

$$24 + 12X = 30$$
$$12X = 6$$
$$X = \$0.50 \text{ per lb}$$

3. $4.75 \div 5 = \$0.95$ per lb

	amount	cost per unit	value
wheat	100	X	100X
corn	50	2X	100X
mixture	150	.95	142.50

$$100X + 100X = 142.50$$
$$200X = 142.50$$
$$X = \$.71 \text{ per lb for wheat}$$
$$X = \$1.43 \text{ per lb for corn}$$

Honors Lesson 30

1.
$$R = (20 + X)(80 - 2X)$$
$$R = -2X^2 + 40X + 1600$$
$$R = \left(-X^2 + 20X + 800\right)(2)$$
$$\frac{-b}{2a} = \frac{-20}{2(-1)} = 10$$
$$20 + 10 = 30 \text{ people}$$
$$R = -2(10)^2 + 40(10) + 1600$$
$$R = -200 + 400 + 1600 = \$1800$$

2. $R = (500 + 20W)(.90 - .03W)$

$R = -.6W^2 + 3W + 450$

$\frac{-b}{2a} = \frac{-3}{2(-.6)} = 2.5$ weeks

$R = -.6(2.5)^2 + 3(2.5) + 450$

$R = -3.75 + 7.5 + 450$

$R = \$453.75$

3. $R = (100{,}000 - 5{,}000X)(3 + .5X)$

$R = -2{,}500X^2 + 35{,}000X + 300{,}000$

$\frac{-b}{2a} = \frac{-35{,}000}{2(-2{,}500)} = 7$ units of .50

$\$3.00 + \$3.50 = \$6.50$

$R = -2{,}500(7)^2 + 35{,}000(7) + 300{,}000$

$R = -122{,}500 + 245{,}000 + 300{,}000$

$R = \$422{,}500$

4. $Y = (30 + X)\left(20 - \frac{1}{3}X\right)$ $\qquad X \le 18$

$Y = -\frac{1}{3}X^2 + 10X + 600$

$\frac{-b}{2a} = \frac{-10}{2\left(-\frac{1}{3}\right)} = 15$ more trees

$30 + 15 = 45$ trees per acre

Honors Lesson 31

1. $-4X + 3Y = 2; \quad -2X + Y = -6$

$$X = \frac{\begin{vmatrix} 2 & 3 \\ -6 & 1 \end{vmatrix}}{\begin{vmatrix} -4 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{2 - (-18)}{-4 - (-6)} = \frac{20}{2} = 10$$

$$Y = \frac{\begin{vmatrix} -4 & 2 \\ -2 & -6 \end{vmatrix}}{\begin{vmatrix} -4 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{24 - (-4)}{-4 - (-6)} = \frac{28}{2} = 14$$

2. $3X - Y = 1;\ 3X + 4Y = -19$

$$X = \frac{\begin{vmatrix} 1 & -1 \\ -19 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 3 & 4 \end{vmatrix}} = \frac{4 - (19)}{12 - (-3)} = \frac{-15}{15} = -1$$

$$Y = \frac{\begin{vmatrix} 3 & 1 \\ 3 & -19 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 3 & 4 \end{vmatrix}} = \frac{-57 - (3)}{12 - (-3)} = \frac{-60}{15} = -4$$

3. $X + 4Y = 11;\ -3X + 2Y = 9$

$$X = \frac{\begin{vmatrix} 11 & 4 \\ 9 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix}} = \frac{22 - (36)}{2 - (-12)} = \frac{-14}{14} = -1$$

$$Y = \frac{\begin{vmatrix} 1 & 11 \\ -3 & 9 \end{vmatrix}}{\begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix}} = \frac{9 - (-33)}{2 - (-12)} = \frac{42}{14} = 3$$

4. $5X - 3Y + 3Z = 3;\ 2X - 6Y - 4Z = 2;\ 3X - 5Y + Z = -3$

$$X = \frac{\begin{vmatrix} 3 & -3 & 3 \\ 2 & -6 & -4 \\ -3 & -5 & 1 \end{vmatrix}}{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & -4 \\ 3 & -5 & 1 \end{vmatrix}} = \frac{(3)(-6)(1) + (-3)(-4)(-3) + (3)(2)(-5) - (-3)(-6)(3) - (-5)(-4)(3) - (1)(2)(-3)}{(5)(-6)(1) + (-3)(-4)(3) + (3)(2)(-5) - (3)(-6)(3) - (-5)(-4)(5) - (1)(2)(-3)}$$

$$\frac{(-18) + (-36) + (-30) - (54) - (60) - (-6)}{(-30) + (36) + (-30) - (-54) - (100) - (-6)} = \frac{-192}{-64} = 3$$

$$Y = \frac{\begin{vmatrix} 5 & 3 & 3 \\ 2 & 2 & -4 \\ 3 & -3 & 1 \end{vmatrix}}{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & -4 \\ 3 & -5 & 1 \end{vmatrix}} = \frac{(5)(2)(1) + (3)(-4)(3) + (3)(2)(-3) - (3)(2)(3) - (-3)(-4)(5) - (1)(2)(3)}{(5)(-6)(1) + (-3)(-4)(3) + (3)(2)(-5) - (3)(-6)(3) - (-5)(-4)(5) - (1)(2)(-3)} =$$

$$\frac{(10) + (-36) + (-18) - (18) - (60) - (6)}{(-30) + (36) + (-30) - (-54) - (100) - (-6)} = \frac{-128}{-64} = 2$$

$$Z = \frac{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & 2 \\ 3 & -5 & -3 \end{vmatrix}}{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & -4 \\ 3 & -5 & 1 \end{vmatrix}} = \frac{(5)(-6)(-3) + (-3)(2)(3) + (3)(2)(-5) - (3)(-6)(3) - (-5)(2)(5) - (-3)(2)(-3)}{(5)(-6)(1) + (-3)(-4)(3) + (3)(2)(-5) - (3)(-6)(3) - (-5)(-4)(5) - (1)(2)(-3)} =$$

$$\frac{(90) + (-18) + (-30) - (-54) - (-50) - (18)}{(-30) + (36) + (-30) - (-54) - (100) - (-6)} = \frac{128}{-64} = -2$$