LESSON 12

Quadratic Formula

A ***quadratic*** is an equation that has an unknown or variable raised to the second power, as in Y^2 or A^2. In factoring and in completing the square, we have been dealing exclusively with quadratic equations. So far, we can find the solution to a quadratic equation by factoring it, or if this fails, by completing the square. In this lesson we are going to complete the square with variables in order to discover a formula to solve all quadratics. If you've mastered the previous lesson, try solving the following equation by completing the square, and then compare your solution with mine.

$$AX^2 + BX + C = 0$$

Divide by the coefficient of X^2.

$$\frac{AX^2}{A} + \frac{BX}{A} + \frac{C}{A} = 0$$

$$X^2 + \frac{BX}{A} + \frac{C}{A} = 0$$

Add the opposite of the third term to both sides.

$$X^2 + \frac{BX}{A} = -\frac{C}{A}$$

Take one-half of the coefficient of the middle term, square it, and add the result to both sides.

$$X^2 + \frac{BX}{A} + \left(\frac{B}{2A}\right)^2 = -\frac{C}{A} + \left(\frac{B}{2A}\right)^2$$

Factor the left side.

$$\left(X + \frac{B}{2A}\right)^2 = -\frac{C}{A} + \frac{B^2}{4A^2}$$

Combine terms on the right.

$$\left(X + \frac{B}{2A}\right)^2 = -\frac{4AC}{4A^2} + \frac{B^2}{4A^2}$$

Take the square root of both sides.

$$X + \frac{B}{2A} = \sqrt{-\frac{4AC}{4A^2} + \frac{B^2}{4A^2}} = \pm\frac{\sqrt{-4AC + B^2}}{2A}$$

Subtract B/2A from both sides, and combine.

$$X = -\frac{B}{2A} \pm \frac{\sqrt{-4AC + B^2}}{2A}$$

The ***quadratic formula***! This is the form in which it is usually written.

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Example 1

Let's try an equation that we can answer by factoring, and "plug in" the values for A, B, and C. Remember that to find A, B, and C, the equation must be in the form $AX^2 + BX + C = 0$.

$X^2 + 5X + 6 = 0$

A = 1, B = 5, and C = 6

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$X = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2}$$

$$X = \frac{-5 \pm 1}{2} = \frac{-4}{2} \text{ or } \frac{-6}{2} = -2 \text{ or } -3$$

We can also solve $X^2 + 5X + 6 = 0$ by factoring.

$$X^2 + 5X + 6 = 0$$
$$(X + 2)(X + 3) = 0$$
$$X + 2 = 0 \quad X + 3 = 0$$
$$X = -2 \qquad X = -3$$

For this problem, it would have much easier to solve by factoring. Try factoring first, and if it doesn't work, use the quadratic formula. Here is another problem to try.

Example 2

Find the factors of $2X^2 = -7X - 4$.

To find A, B, and C, the equation must be in the form $AX^2 + BX + C = 0$.

$$2X^2 + 7X + 4 = 0$$

A = 2, B = 7, and C = 4

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$
$$X = \frac{-7 \pm \sqrt{49 - 32}}{4} = \frac{-7 \pm \sqrt{17}}{4}$$
$$X = \frac{-7 \pm \sqrt{17}}{4}$$
$$X = \frac{-7 + \sqrt{17}}{4} \text{ or } \frac{-7 - \sqrt{17}}{4}$$

Practice Problems 1

Solve for X. Try factoring first, and then use the quadratic formula if necessary.

1. $X^2 - 25 = 0$
2. $X^2 - 18X = -81$
3. $2X^2 + 7X + 6 = 0$
4. $3X^2 + X - 4 = 0$
5. $4A^2 - 36 = 0$
6. $X^2 + 5 = -3X$
7. $7X^2 = -2X + 1$
8. $2X^2 + 2X - 5 = 0$
9. $\frac{5}{X+3} + \frac{2}{X-3} = 5 \quad (X \neq \pm 3)$
10. $4X^2 = 9$
11. $4X^2 + 20X = -25$
12. $3Q^2 = -4Q - 2$

Solutions 1

1. $(X+5)(X-5) = 0$

   $X+5=0 \quad X-5=0$

   $X=-5 \quad X=5$

2. $(X-9)(X-9) = 0$

   $X-9=0 \quad X-9=0$

   $X=9 \quad X=9$

3. $(2X+3)(X+2) = 0$

   $2X+3=0 \quad X+2=0$

   $2X=-3$

   $X=-3/2 \quad X=-2$

4. $(3X+4)(X-1) = 0$

   $3X+4=0 \quad X-1=0$

   $3X=-4$

   $X=-4/3 \quad X=1$

5. $(2A-6)(2A+6) = 0$

   $2A-6=0 \quad 2A+6=0$

   $2A=6 \quad 2A=-6$

   $A=6/2 \quad A=-6/2$

   $A=3 \quad A=-3$

6. $X = \dfrac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$

$X = \dfrac{-3 \pm \sqrt{-11}}{2} = \dfrac{-3 + i\sqrt{11}}{2}$ or $X = \dfrac{-3 - i\sqrt{11}}{2}$

7. $X = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 7 \cdot -1}}{2 \cdot 7} = \dfrac{-2 \pm 4\sqrt{2}}{14} = \dfrac{-1 + 2\sqrt{2}}{7}$ or $X = \dfrac{-1 - 2\sqrt{2}}{7}$

8. $X = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2} = \dfrac{-2 \pm 2\sqrt{11}}{4} = \dfrac{-1 + \sqrt{11}}{2}$ or $X = \dfrac{-1 - \sqrt{11}}{2}$

9. $\left(\dfrac{5}{X+3} + \dfrac{2}{X-3}\right) = 5$

$5(X-3) + 2(X+3) = 5(X^2 - 9)$

$7X - 9 = 5X^2 - 45$

$5X^2 - 7X - 36 = 0$

$X = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 5 \cdot -36}}{2 \cdot 5}$

$X = \dfrac{7 \pm \sqrt{769}}{10}$

$X = \dfrac{7 + \sqrt{769}}{10}$ or $X = \dfrac{7 - \sqrt{769}}{10}$

10. $(2X-3)(2X+3) = 0$

$2X - 3 = 0$ $\quad$ $2X + 3 = 0$

$2X = 3$ $\quad$ $2X = -3$

$X = 3/2$ $\quad$ $X = -3/2$

11. $(2X+5)(2X+5) = 0$

$2X + 5 = 0$ $\quad$ $2X + 5 = 0$

$2X = -5$ $\quad$ $2X = -5$

$X = -5/2$ $\quad$ $X = -5/2$

12. $3Q^2 + 4Q + 2 = 0$ $\quad$ $X = \dfrac{-(4) \pm \sqrt{(4)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3}$

$X = \dfrac{-4 \pm \sqrt{16 - 24}}{2 \cdot 3} = \dfrac{-4 \pm \sqrt{-8}}{2 \cdot 3}$

$X = \dfrac{-4 \pm i\sqrt{2 \cdot 4}}{2 \cdot 3} = \dfrac{-\cancel{4} \pm \cancel{2}i\sqrt{2}}{\cancel{2} \cdot 3}$

$X = \dfrac{-2 + i\sqrt{2}}{3}$ or $\dfrac{-2 - i\sqrt{2}}{3}$