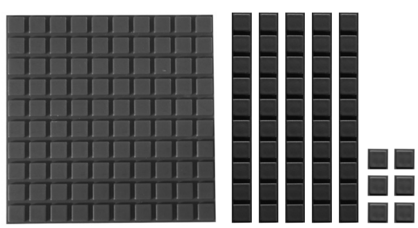
LESSON 20

Addition and Multiplication of Polynomials

Base 10 and Base X

Recall the factors of each of the pieces in base 10. The unit block (green) is 1 x 1.

The 10 bar (blue) is 1 x 10, and the 100 square (red) is 10 x 10. Each of these pieces may also be expressed in terms of exponents: 1 x 1 = 1, which is 1^0; 1 x 10 = 10, which is 10^1; 10 x 10 = 100, which is 10^2. Below is the number 156 shown with the blocks and expressed in different ways.



100 + 50 + 6

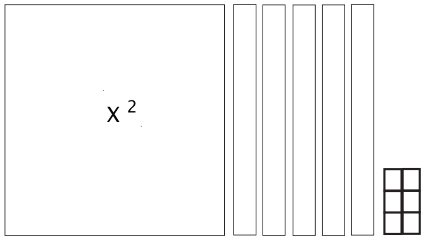
1 x 100 + 5 x 10 + 6 x 1

$1 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

In the decimal system, every value is based on 10. The decimal system is referred to as base 10.

In algebra, the unit bar is still one by one. The smooth blue piece that snaps into the back of the 10 bar is one by X, and the smooth red piece that snaps into the back of the 100 square is X by X. Each of these pieces may also be expressed in terms of exponents: $1 \cdot 1 = 1$, which is 1^0, or X^0 (which is the same thing since both are equal to one); $1 \cdot X = X$, which is X^1; and $X \cdot X = X^2$.

On the next page is the polynomial $X^2 + 5X + 6$ shown with the blocks and expressed in different ways.



$$\begin{array}{ccccc} X^2 & + & 5X & + & 6 \\ X \cdot X & + & 5 \cdot X & + & 6 \cdot 1 \\ 1 \cdot X^2 & + & 5 \cdot X^1 & + & 6 \cdot X^0 \end{array}$$

In algebra, every value is based on X. Algebra is arithmetic in base X.

Kinds of Polynomials

Polynomial derives from polys (many) and nomen (name), so literally it means "many names." If a polynomial has three components, it is called a *trinomial* (tri- meaning "three"). A *binomial* (bi- meaning "two") has two parts. A *monomial* (mono- meaning "one") has one part.

In the next few sections, whenever you feel the need to reassure yourself that you are on the right track, simply change the equation from base X to base 10, and redo the problem.

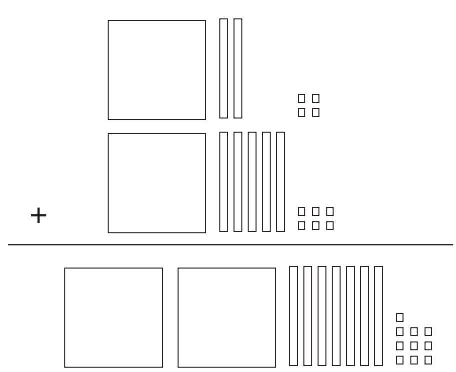
On the next two pages, operations that you are familiar with, such as addition and multiplication, will now be performed with polynomials in base X instead of base 10. Take your time and remember the connection with what you already know. Someone has said, "Algebra is not difficult, just different."

Addition of Polynomials

When adding or subtracting polynomials, remember that "to combine, they must be the same kind." Units may be added (or subtracted) with other units, Xs with Xs, X^2s with X^2s, etc. Since we don't know what the value is for X, all the addition and subtraction is done in the coefficients. Read through the following examples for clarity. The gray inserts are used to show –X.

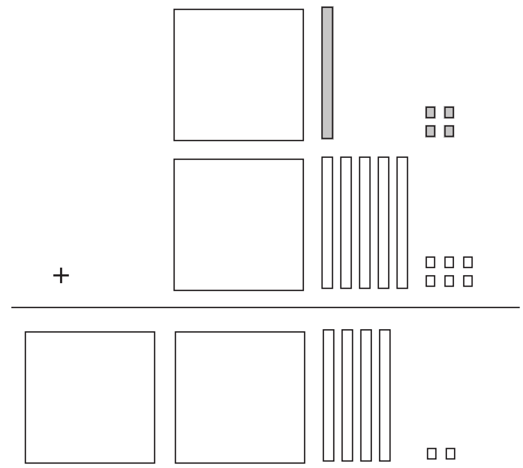
Example 1

$$\begin{array}{r} X^2 + 2X + 4 \\ + X^2 + 5X + 6 \\ \hline 2X^2 + 7X + 10 \end{array}$$



Example 2

$$\begin{array}{r} X^2 - \ X - 4 \\ + X^2 + 5X + 6 \\ \hline 2X^2 + 4X + 2 \end{array}$$



Multiplication of Polynomials

When we multiply two binomials, the result is a trinomial. I like to use the same format for multiplying a binomial as for multiplying any double-digit number in the decimal system.

Let's look at a problem in the decimal system using expanded notation.

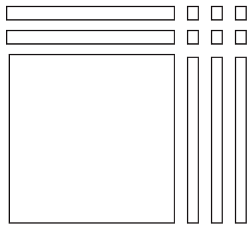
Example 3

$$\begin{array}{r} 13 \rightarrow \\ \times 12 \uparrow \\ \hline \end{array} = \begin{array}{r} 10 + 3 \\ \times 10 + 2 \\ \hline 20 + 6 \\ 100 + 30 \quad \\ \hline 100 + 50 + 6 \end{array}$$

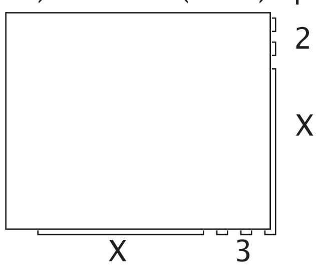
If this were in base X instead of base 10, it would look like this:

$$\begin{array}{r} X + 3 \\ \times X + 2 \\ \hline 2X + 6 \\ X^2 + 3X \quad \\ \hline X^2 + 5X + 6 \end{array}$$

The area, or product, of this rectangle is $X^2 + 5X + 6$. Do you see it?

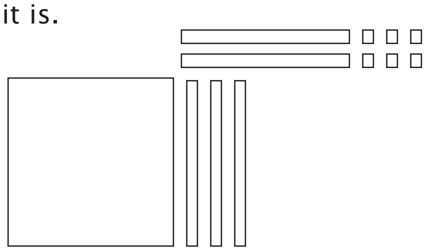


In this rectangle, we cover up most of it to reveal the factors, which are $(X + 3)$ over and $(X + 2)$ up.



The written equivalent of the picture looks just like double-digit multiplication, which it is.

$$\begin{array}{r} X+3 \rightarrow \\ \underline{\times X+2} \uparrow \\ 2X+6 \\ \underline{X^2+3X} \\ X^2+5X+6 \end{array}$$



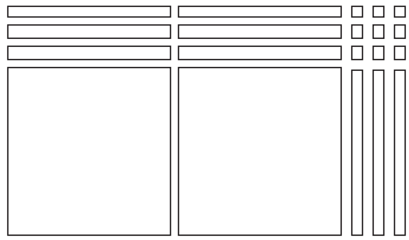
Example 4

$$\begin{array}{r} 23 \rightarrow \\ \underline{\times 13} \uparrow \end{array} = \begin{array}{r} 20+3 \\ \underline{\times 20+3} \\ 60+9 \\ \underline{200+30} \\ 200+90+9 \end{array}$$

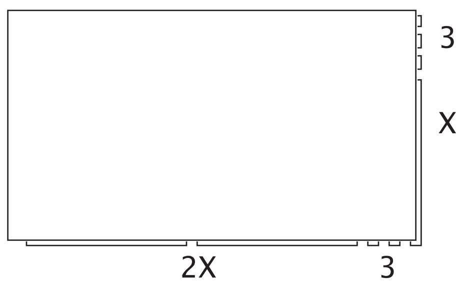
If this were in base X instead of base 10, it would look like this:

$$\begin{array}{r} 2X+3 \\ \underline{\times X+3} \\ 6X+9 \\ \underline{2X^2+3X} \\ 2X^2+9X+9 \end{array}$$

The area, or product, of this rectangle is $2X^2 + 9X + 9$. Do you see it?

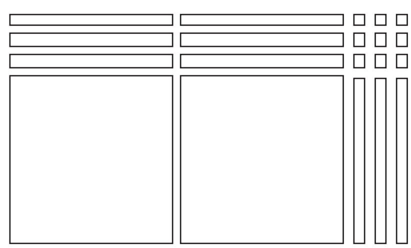


In this rectangle, we cover up most of it to reveal the factors, which are $(2X + 3)$ over and $(X + 3)$ up.

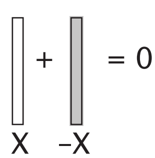


The written equivalent of the picture looks just like double-digit multiplication, which it is.

$$\begin{array}{r} 2X+3 \rightarrow \\ \times\ X+3 \uparrow \\ \hline 6X+9 \\ 2X^2+3X \quad \\ \hline 2X^2+9X+9 \end{array}$$

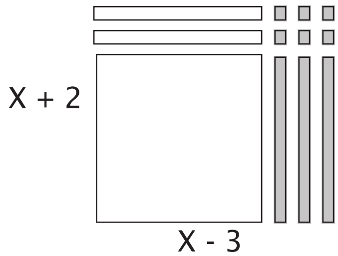


You can also multiply binomials that include negative numbers. To show $-X$, use the gray inserts. The addition identity tells us that $X + (-X) = 0$.



Example 5

$$\begin{array}{r} X-3 \rightarrow \\ \times\, X+2 \uparrow \\ \hline 2X-6 \\ X^2-3X \quad \\ \hline X^2-\ \ X-6 \end{array}$$



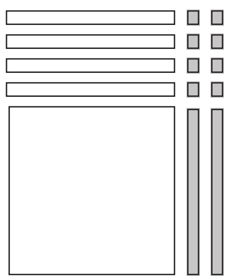
Here it is in base 10:

$$\begin{array}{rl} 10-3 & =7 \\ \times\, 10+2 & =12 \\ \hline 20-6 & \ 84 \\ 100-30 \quad & \ \nwarrow\!\searrow \\ \hline 100-10-6 & =84 \end{array}$$

Look at the next two examples carefully.

Example 6

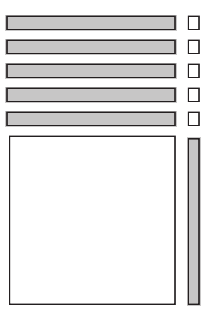
(X – 2)(X + 4)



$$\begin{array}{r} X-2 \rightarrow \\ \times\, X+4 \uparrow \\ \hline 4X-8 \\ X^2-2X \quad \\ \hline X^2-2X-8 \end{array}$$

Example 7

(X – 1)(X – 5)



$$\begin{array}{r} X-1 \rightarrow \\ \times\, X-5 \uparrow \\ \hline -5X+5 \\ X^2-\ \ X \quad \\ \hline X^2-6X+5 \end{array}$$